

Mapping the electron gas temperature across the entire sky through the relativistic SZ effect

Mathieu Remazeilles

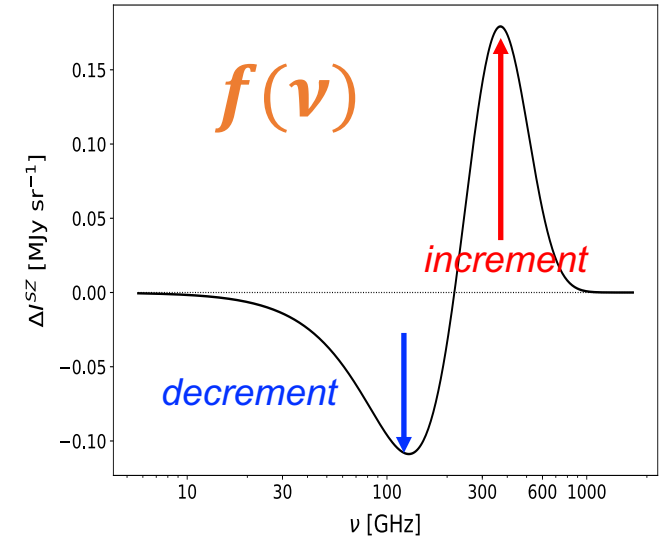
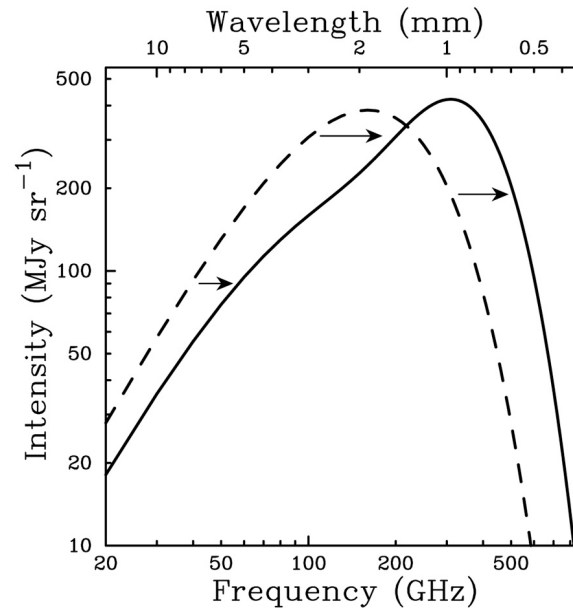
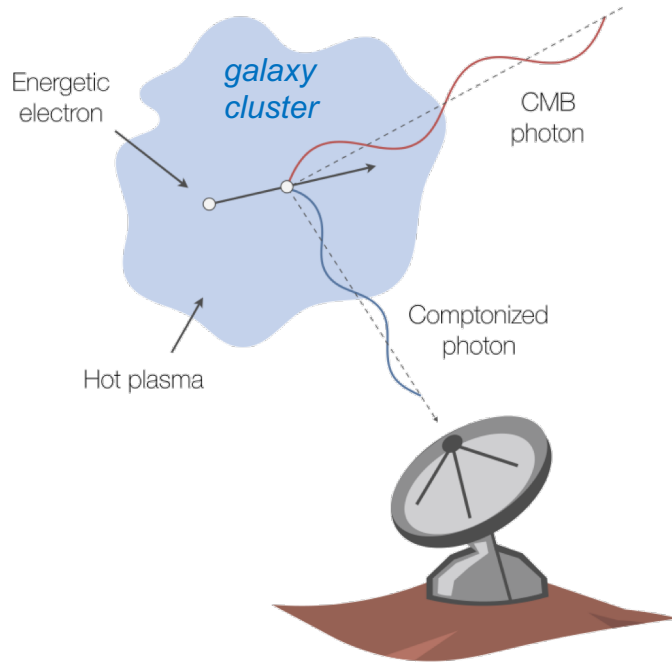


The University of Manchester

Remazeilles & Chluba
[arXiv:1907.00916](https://arxiv.org/abs/1907.00916)

Millimetron Workshop 2019
Paris, 9-11 Sep 2019

Thermal Sunyaev-Zeldovich (SZ) effect



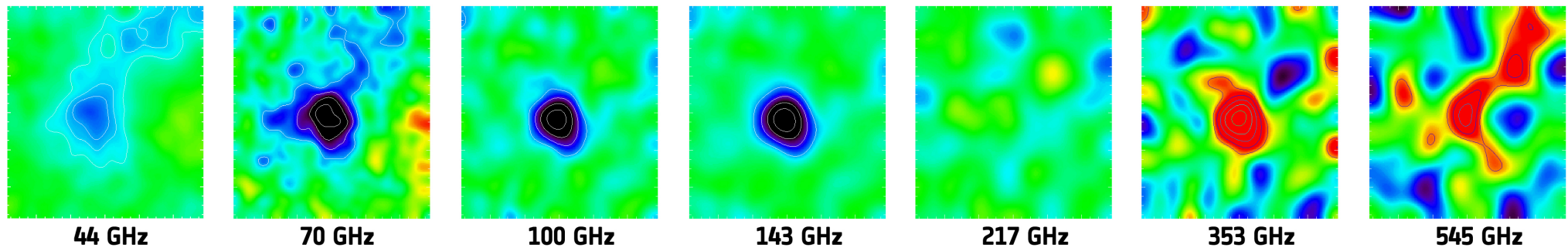
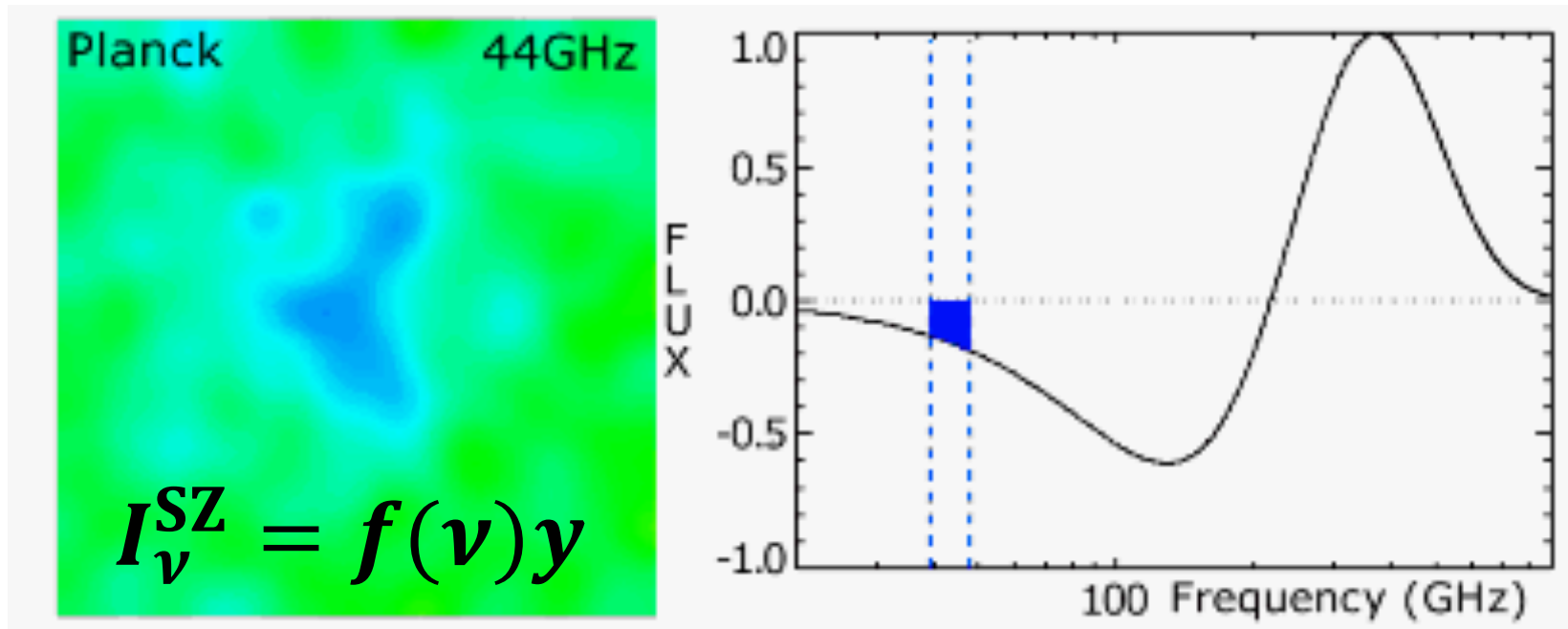
Inverse Compton scattering of CMB photons by the hot gas of electrons in galaxy clusters

y-type spectral distortion of the CMB blackbody spectrum

Spectral signature

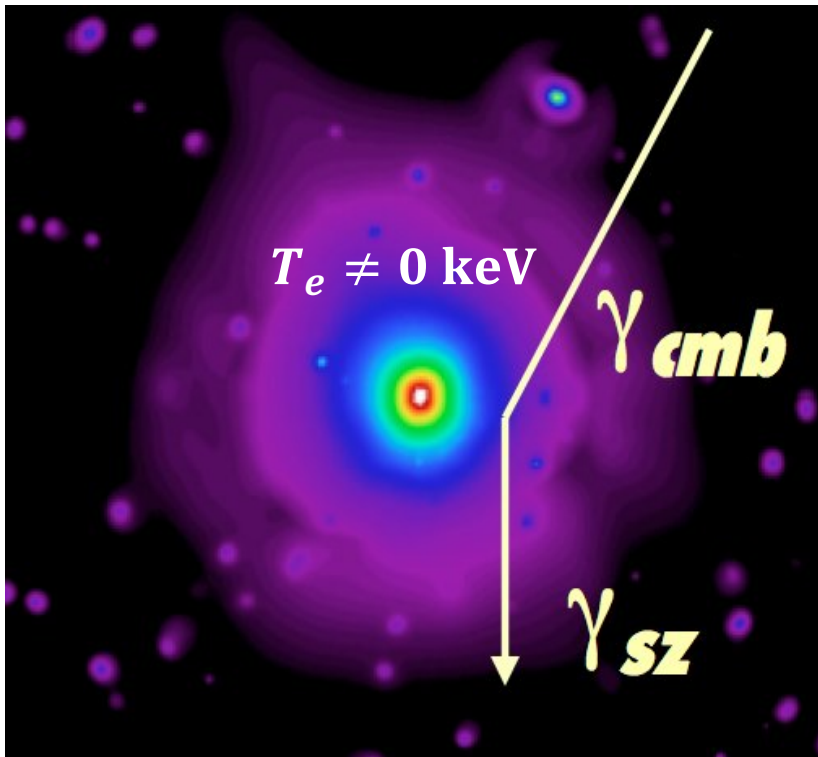
$$I_{\nu}^{\text{SZ}} \equiv \frac{\Delta I_{\nu}^{\text{CMB}}}{I_{\nu}^{\text{CMB}}} = f(\nu) \frac{\sigma_T}{m_e c^2} \int P_e(l) dl = f(\nu) y$$

Spectroscopy of clusters across frequencies



Credit: ESA/Planck Collaboration

rSZ: relativistic temperature corrections to the thermal SZ effect



In hot galaxy clusters, **relativistic temperature corrections** to the thermal SZ effect should be accounted for:

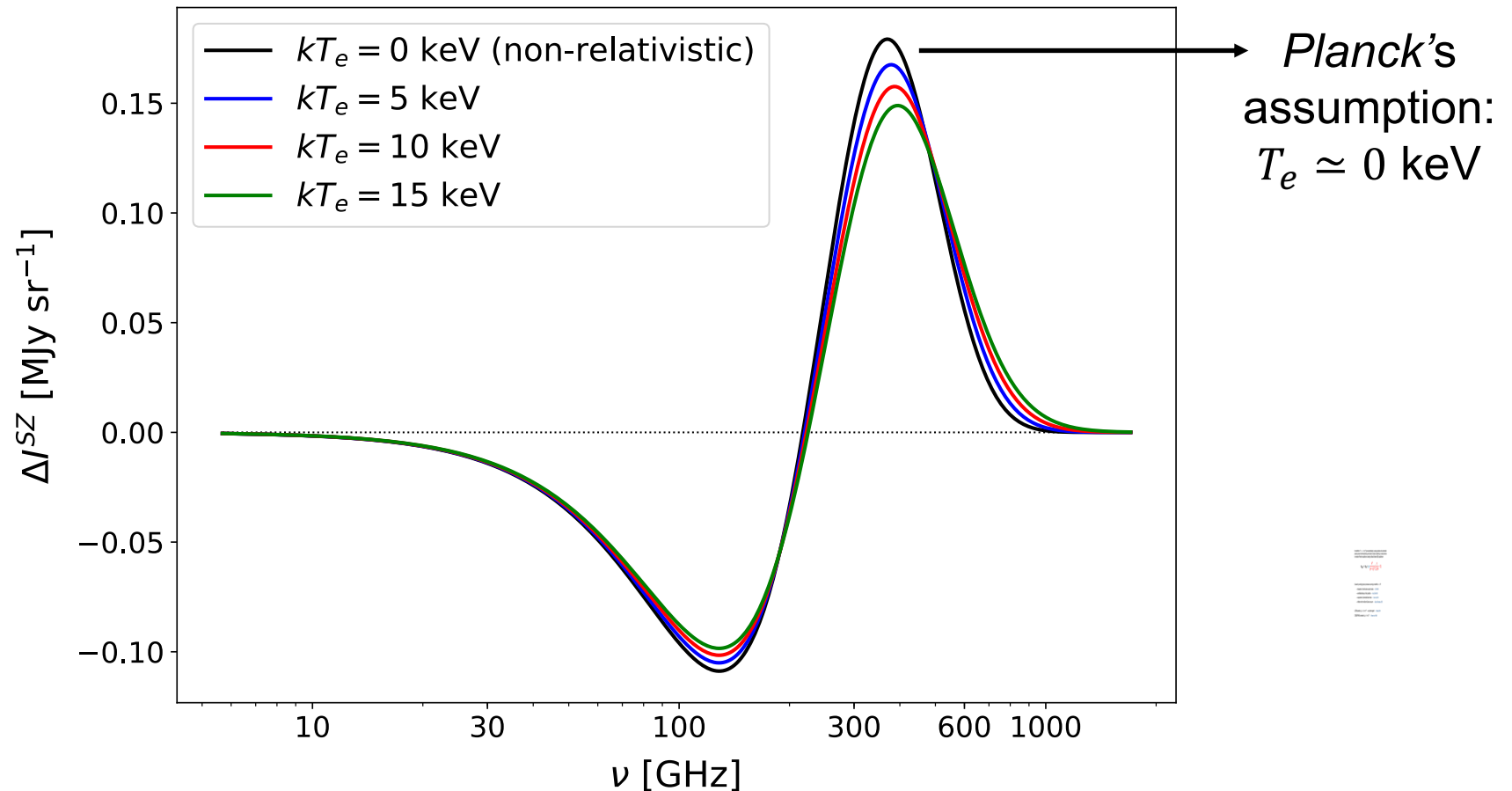
$$\begin{aligned} I^{SZ}(\nu, \vec{\theta}) &= f(\nu, T_e) y(\vec{\theta}) \\ &= (f(\nu) + \delta(\nu, T_e)) y(\vec{\theta}) \end{aligned}$$

f(ν): non-relativistic spectrum ($T_e \simeq 0$ keV)

The spectral signature of SZ emission from galaxy clusters changes with the local electron gas temperature

Relativistic temperature corrections to SZ

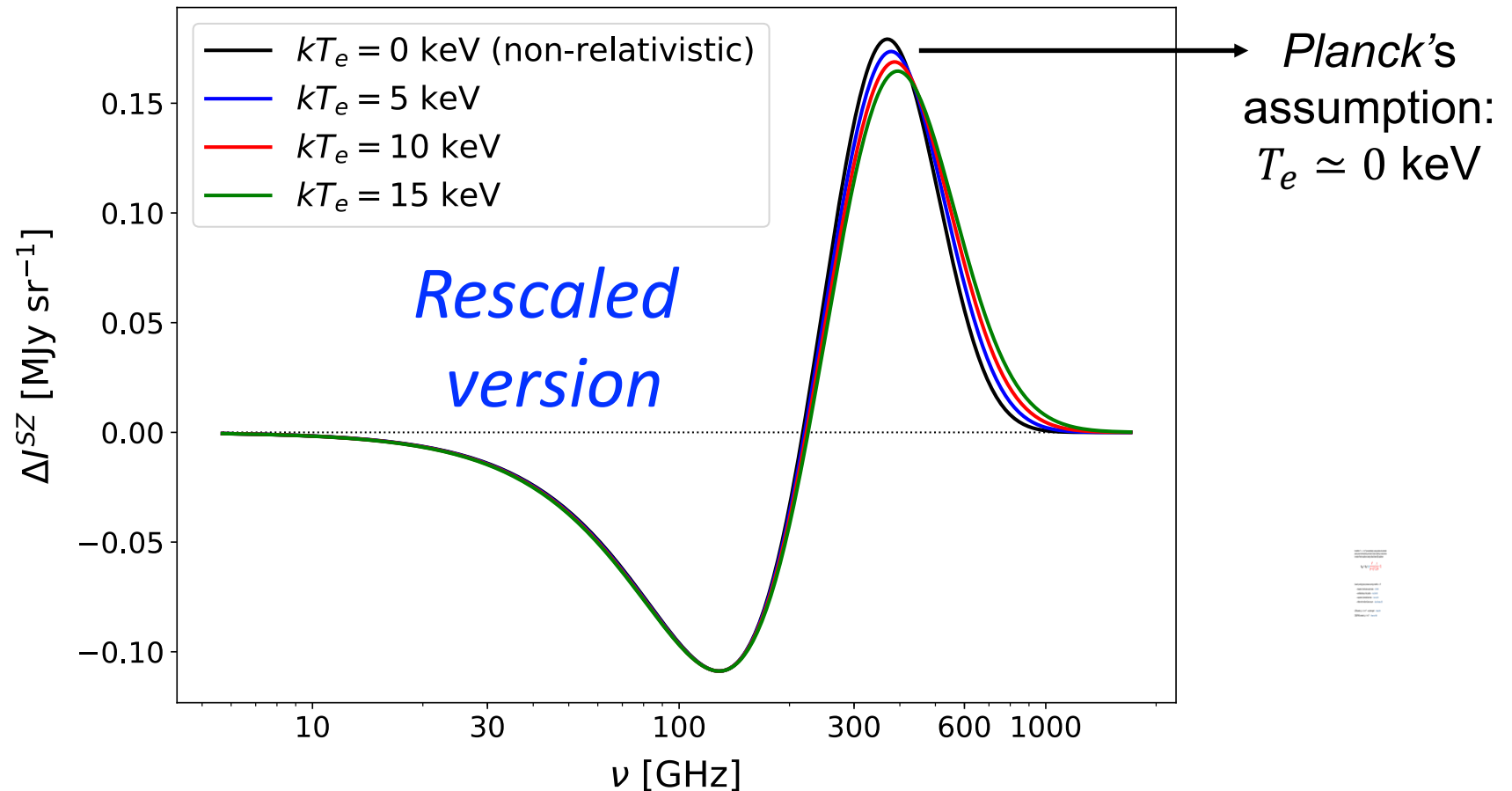
$$I^{SZ}(\nu, \vec{\theta}) = f(\nu, T_e) y(\vec{\theta})$$



The spectral signature of SZ emission from galaxy clusters changes with the local electron gas temperature

Relativistic temperature corrections to SZ

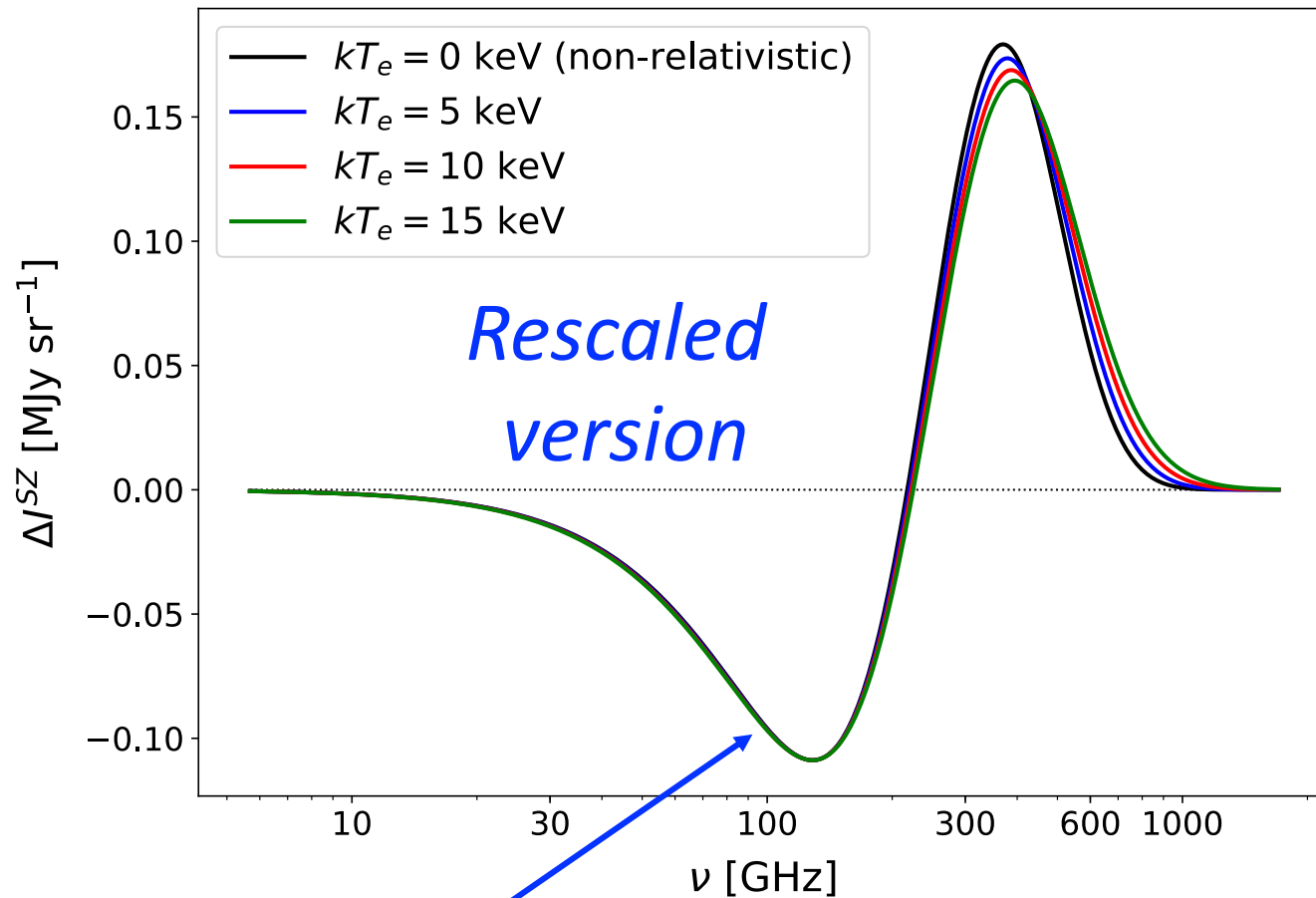
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The spectral signature of SZ emission from galaxy clusters changes with the local electron gas temperature

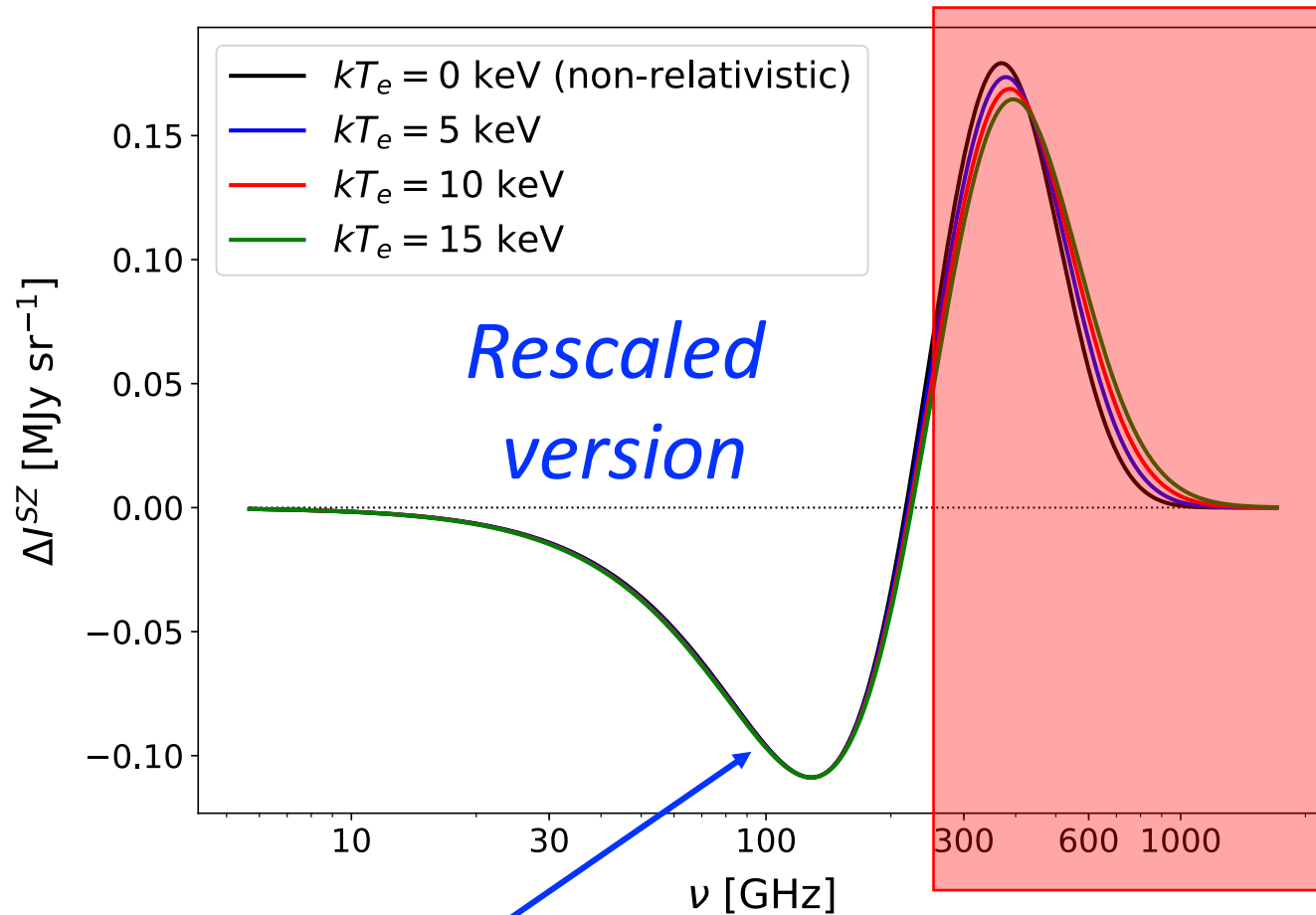
High frequencies are crucial for rSZ !

$$I^{SZ}(\nu, \vec{\theta}) = f(\nu, T_e) y(\vec{\theta})$$



High frequencies are crucial for rSZ !

$$I^{SZ}(\nu, \vec{\theta}) = f(\nu, T_e) y(\vec{\theta})$$

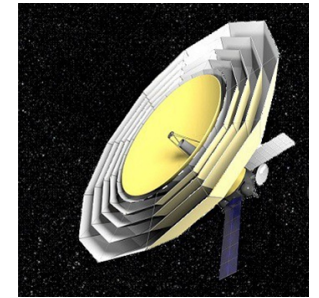
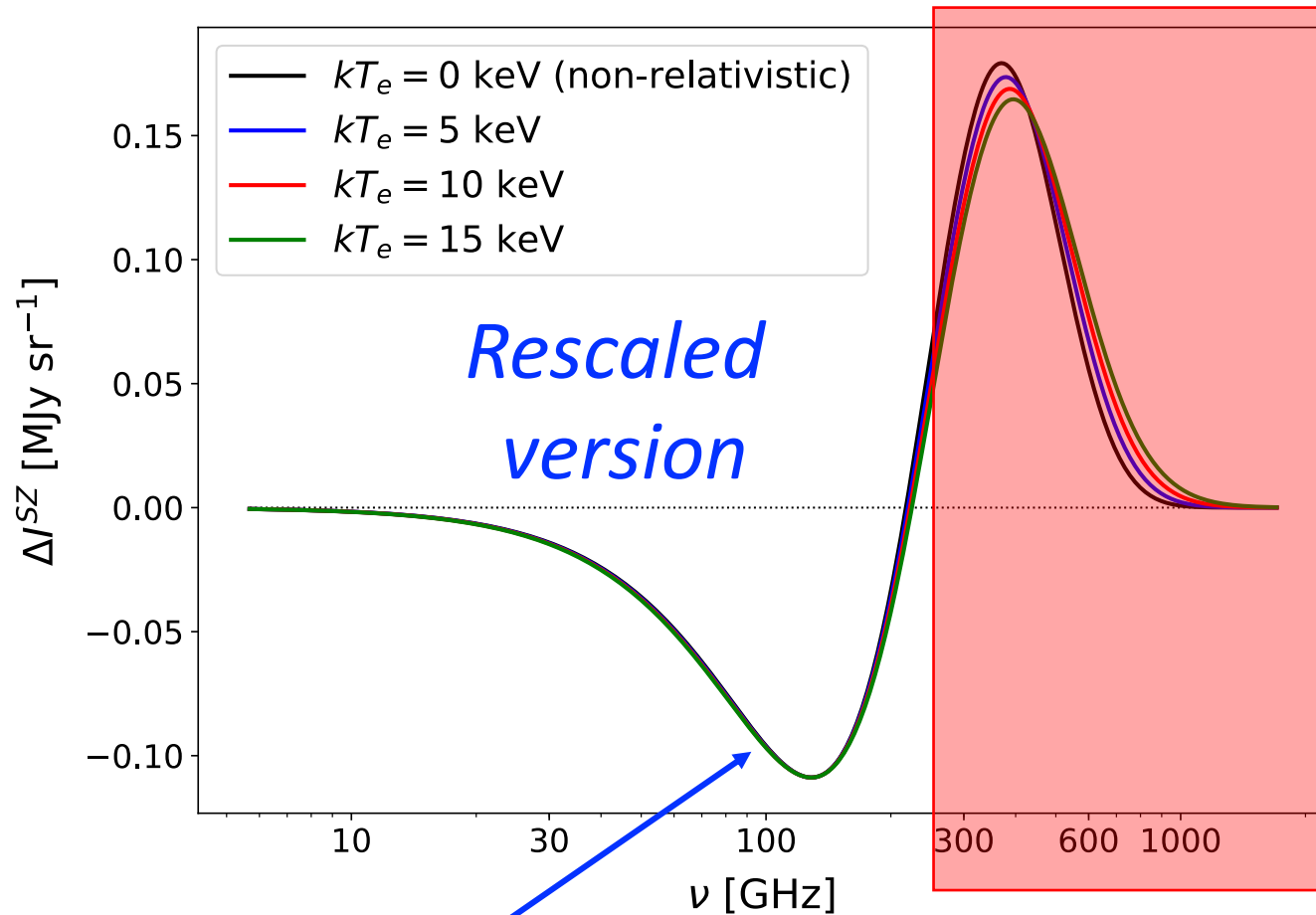


*Spectral shapes are degenerate
at low frequencies*

*High frequencies
needed to extract rSZ*

High frequencies are crucial for rSZ !

$$I^{SZ}(\nu, \vec{\theta}) = f(\nu, T_e) y(\vec{\theta})$$

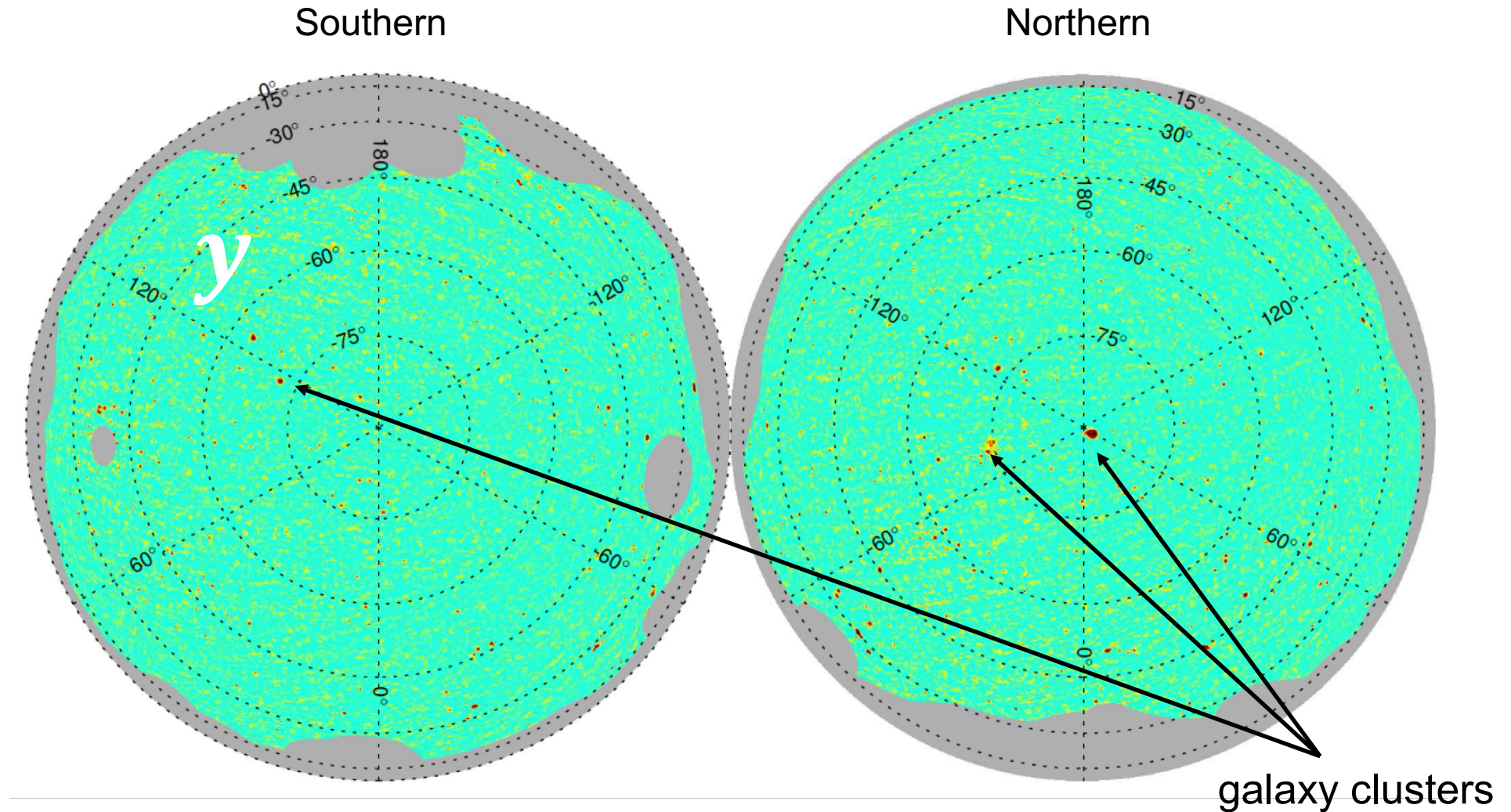


**Millimetron:
great value
for rSZ !**

*Spectral shapes are degenerate
at low frequencies*

*High frequencies
needed to extract rSZ*

First SZ revolution (2010-2020): The *Planck* SZ Compton y -map

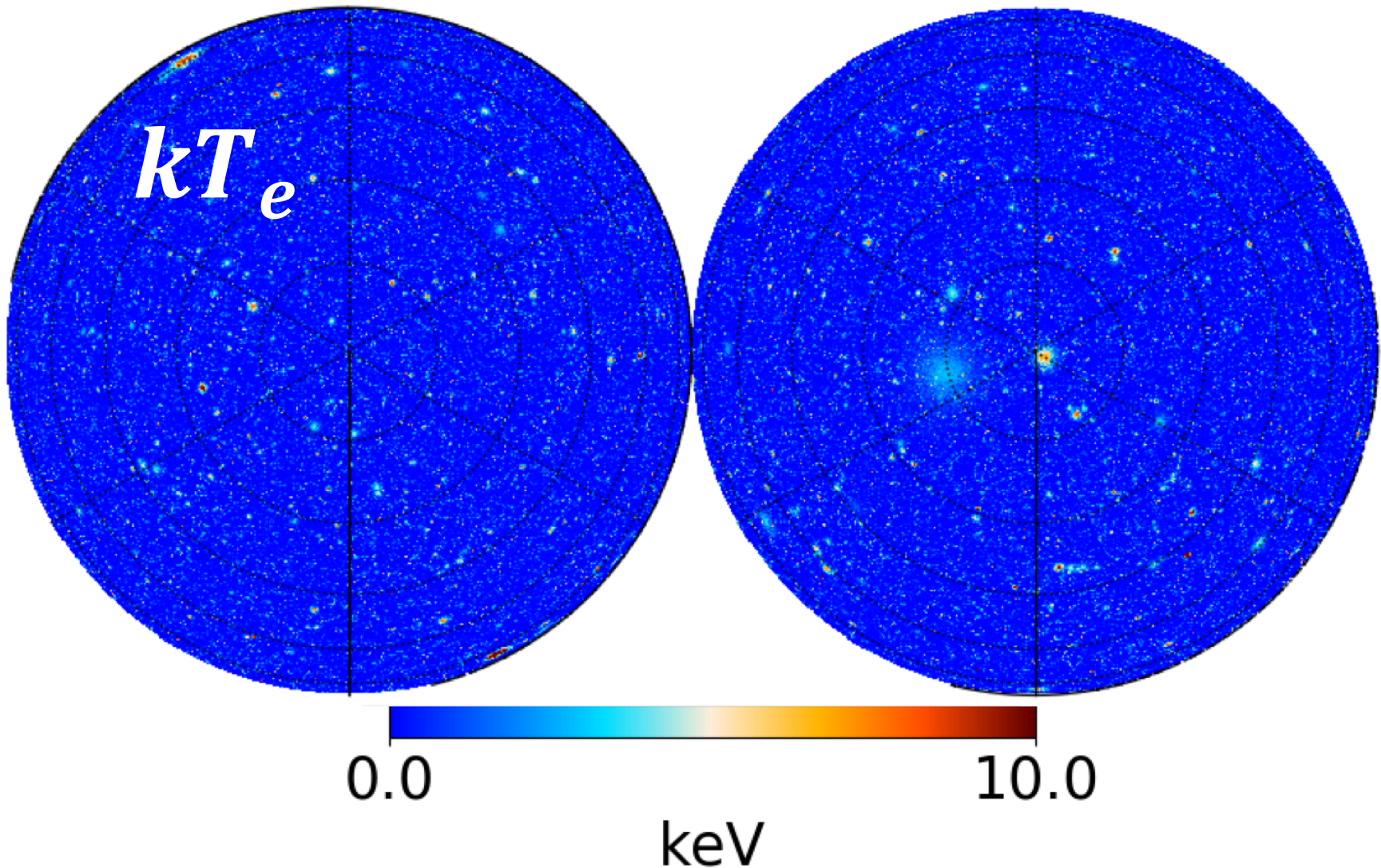


Planck 2015 results XXII, A&A (2016)

Next SZ revolution (2020-2030): The electron temperature T_e -map ?

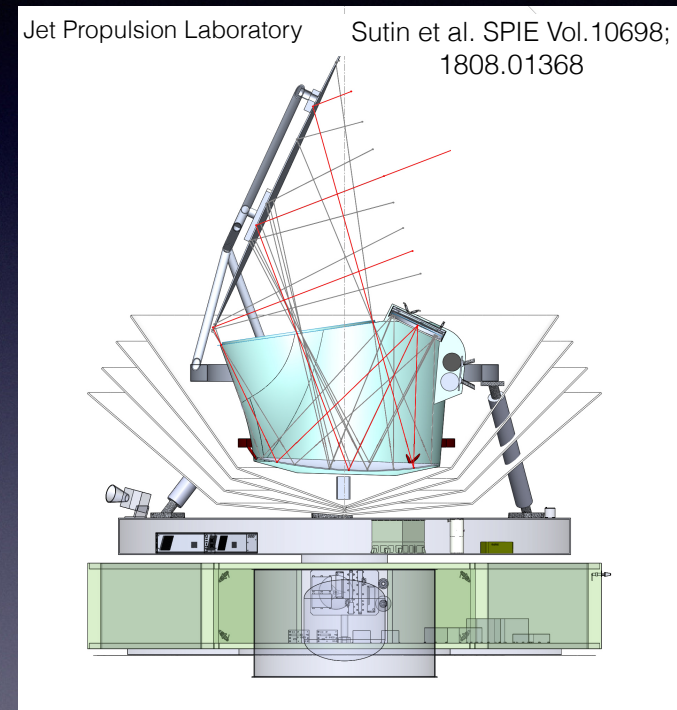
Southern

Northern



PICO in Brief

- Millimeter/submillimeter-wave, polarimetric survey of the entire sky
- 21 bands between 20 GHz and 800 GHz
- 1.4 m aperture telescope
- Diffraction limited resolution: 38' to 1'
- 13,000 transition edge sensor bolometers
- 5 year survey from L2
- 0.87 $\mu\text{K} \cdot \text{arcmin}$ requirement; 0.61 $\mu\text{K} \cdot \text{arcmin}$ goal (=CBE)

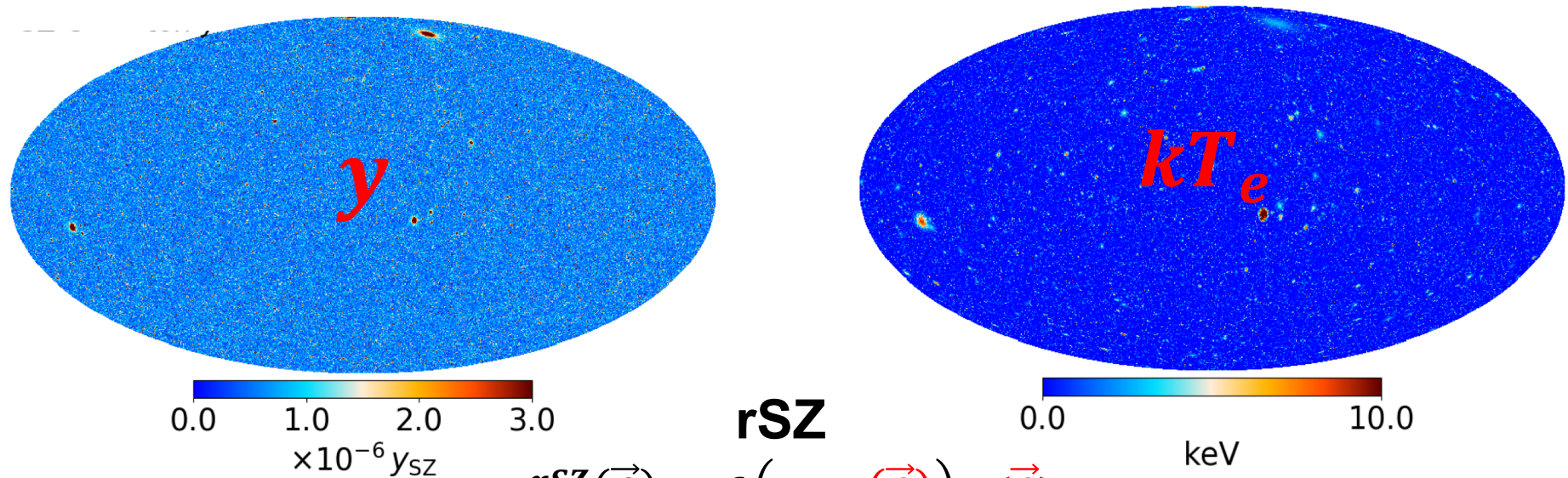


NASA Probe-class Mission Concept Study

(PI: Shaul Hanany)

[arXiv:1902.10541](https://arxiv.org/abs/1902.10541)

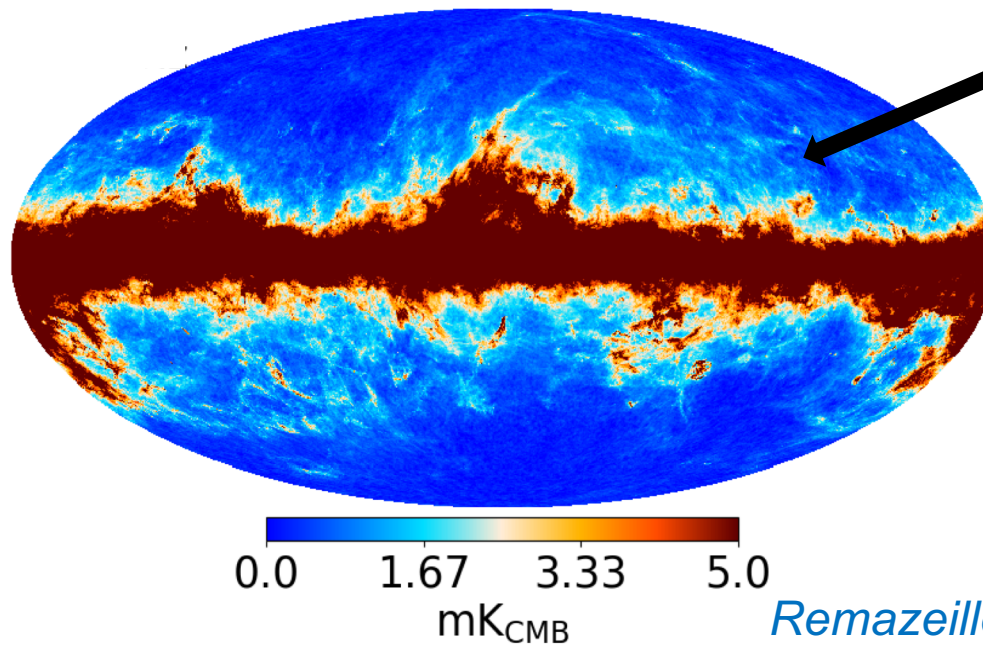
Foreground-obscured sky simulations



rSZ

$$I_\nu^{rSZ}(\vec{\theta}) = f(\nu, T_e(\vec{\theta})) y(\vec{\theta})$$

PICO sky maps



rSZ, kSZ, CMB, CIB,
Galactic foregrounds
(dust, synchrotron,
AME, free-free),
noise

Component Separation

How to disentangle
the y and T_e observables
in sky observations?

SZ-temperature moment expansion

Intensity of relativistic thermal SZ:

$$I_{\nu}^{tSZ}(\vec{\theta}) = f(\nu, T_e(\vec{\theta})) y(\vec{\theta})$$

SZ-temperature moment expansion

Intensity of relativistic thermal SZ:

$$I_{\nu}^{tSZ}(\vec{\theta}) = f(\nu, T_e(\vec{\theta})) y(\vec{\theta})$$

Moment expansion around some pivot temperature $\bar{T}_e$:

$$I_{\nu}^{tSZ}(\vec{\theta}) = f(\nu, \bar{T}_e) y(\vec{\theta}) + \frac{\partial f(\nu, \bar{T}_e)}{\partial T_e} (T_e(\vec{\theta}) - \bar{T}_e) y(\vec{\theta}) + \mathcal{O}(T_e^2)$$

SZ-temperature moment expansion

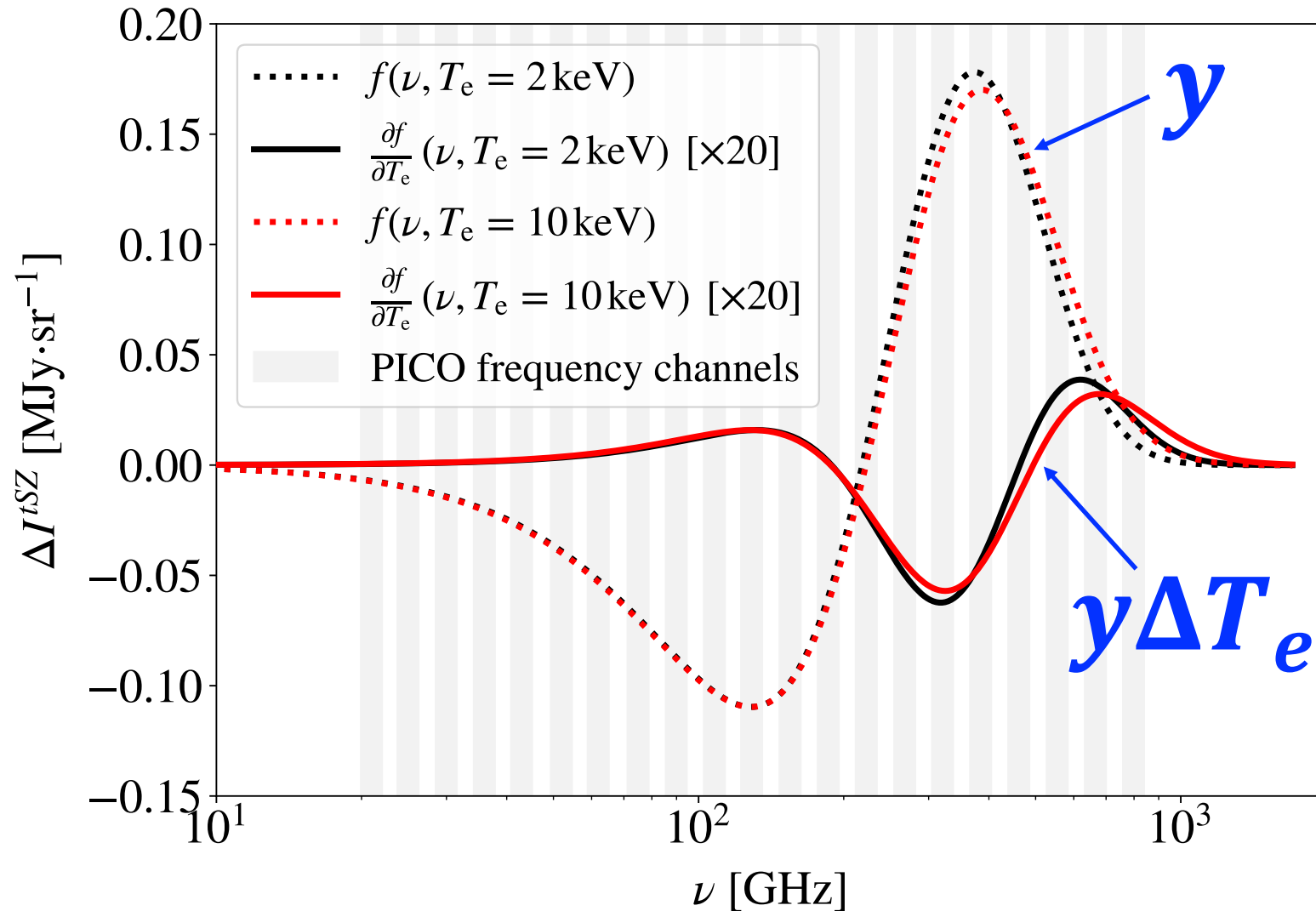
Intensity of relativistic thermal SZ:

$$I_{\nu}^{tSZ}(\vec{\theta}) = f(\nu, T_e(\vec{\theta})) y(\vec{\theta})$$

Moment expansion around some pivot temperature $\bar{T}_e$:

$$I_{\nu}^{tSZ}(\vec{\theta}) = \underbrace{f(\nu, \bar{T}_e)}_{\substack{\text{Spectrum of} \\ y}} \underbrace{y(\vec{\theta})}_{\substack{y \\ \text{component}}} + \underbrace{\frac{\partial f(\nu, \bar{T}_e)}{\partial T_e}}_{\substack{\text{Spectrum of} \\ y\Delta T_e}} \underbrace{(T_e(\vec{\theta}) - \bar{T}_e)y(\vec{\theta})}_{\substack{y\Delta T_e \\ \text{component}}} + \mathcal{O}(T_e^2)$$

Two components of the rSZ effect with distinct spectral signatures



It is possible in principle to disentangle y and $y\Delta T_e$ through multi-frequency observations and component separation methods

rSZ component separation

$$\underbrace{d(\nu, \vec{\theta})}_{\text{image}} = \underbrace{f(\nu, T_e = \bar{T}_e)}_{\text{spectrum of } y} \underbrace{y(\vec{\theta})}_y + \underbrace{\frac{\partial f(\nu, T_e = \bar{T}_e)}{\partial T_e}}_{\text{spectrum of } y\Delta T_e} \underbrace{(T_e(\vec{\theta}) - \bar{T}_e)y(\vec{\theta})}_{y\Delta T_e} + \underbrace{n(\nu, \vec{\theta})}_{\text{foregrounds + noise}}$$

- Component separation with the "Constrained-ILC" ([Remazeilles et al MNRAS 2011](#)):

$$\widehat{y\Delta T_e}(\vec{\theta}) = \sum_{\nu} w(\nu) d(\nu, \vec{\theta}) \quad \text{such that} \quad \begin{cases} \langle \widehat{y\Delta T_e}^2 \rangle \text{ of minimum variance} \\ \sum_{\nu} w(\nu) \partial f(\nu, \bar{T}_e) / \partial T_e = 1 \\ \sum_{\nu} w(\nu) f(\nu, \bar{T}_e) = 0 \end{cases}$$

- Analytic solution: $\mathbf{w} = \frac{(f^t C^{-1} f)(\partial_{T_e} f)^t C^{-1} - ((\partial_{T_e} f)^t C^{-1} f) f^t C^{-1}}{((\partial_{T_e} f)^t C^{-1} \partial_{T_e} f)(f^t C^{-1} f) - ((\partial_{T_e} f)^t C^{-1} f)^2}$ ($C \equiv \langle dd^t \rangle$) yields to:

$$\Rightarrow \widehat{y\Delta T_e}(\vec{\theta}) = (w \cdot f) y(\vec{\theta}) + (w \cdot \frac{\partial f}{\partial \bar{T}_e})(T_e(\vec{\theta}) - \bar{T}_e) y(\vec{\theta}) + w \cdot n$$

rSZ component separation

$$\underbrace{d(\nu, \vec{\theta})}_{\text{image}} = \underbrace{f(\nu, T_e = \bar{T}_e)}_{\text{spectrum of } y} \underbrace{y(\vec{\theta})}_y + \underbrace{\frac{\partial f(\nu, T_e = \bar{T}_e)}{\partial T_e}}_{\text{spectrum of } y\Delta T_e} \underbrace{(T_e(\vec{\theta}) - \bar{T}_e)y(\vec{\theta})}_{y\Delta T_e} + \underbrace{n(\nu, \vec{\theta})}_{\text{foregrounds + noise}}$$

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$$\Rightarrow \widehat{y\Delta T_e}(\vec{\theta}) = \underbrace{(w \cdot f)}_{=0} y(\vec{\theta}) + \underbrace{(w \cdot \frac{\partial f}{\partial T_e})}_{=1} (T_e(\vec{\theta}) - \bar{T}_e) y(\vec{\theta}) + \underbrace{w \cdot n}_{\text{minimum variance}}$$

rSZ component separation

$$\underbrace{d(\nu, \vec{\theta})}_{\text{image}} = \underbrace{f(\nu, T_e = \bar{T}_e)}_{\text{spectrum of } y} \underbrace{y(\vec{\theta})}_y + \underbrace{\frac{\partial f(\nu, T_e = \bar{T}_e)}{\partial T_e}}_{\text{spectrum of } y\Delta T_e} \underbrace{(T_e(\vec{\theta}) - \bar{T}_e)y(\vec{\theta})}_{y\Delta T_e} + \underbrace{n(\nu, \vec{\theta})}_{\text{foregrounds + noise}}$$

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$$\Rightarrow \widehat{y\Delta T_e}(\vec{\theta}) = \underline{(T_e(\vec{\theta}) - \bar{T}_e) y(\vec{\theta})} + \mathbf{w} \cdot \mathbf{n}$$

Temperature-modulated y-map over the sky

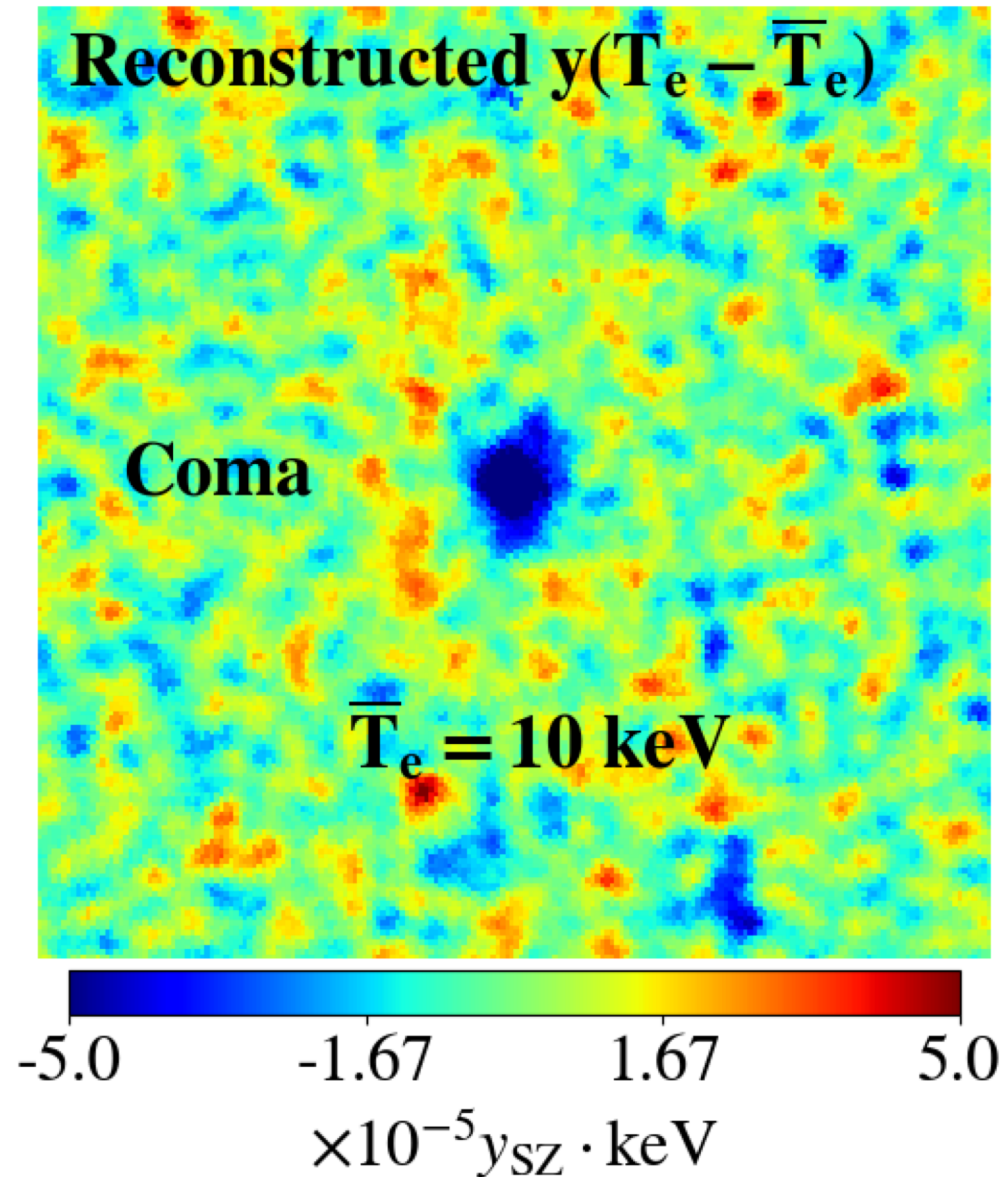
Why is this new SZ component so interesting?

$$y\Delta T_e \equiv y(\vec{\theta})(T_e(\vec{\theta}) - \bar{T}_e)$$

Adopting different pivot temperatures $\bar{T}_e$ in the analysis allows us to conduct a real **temperature spectroscopy** of the cluster:

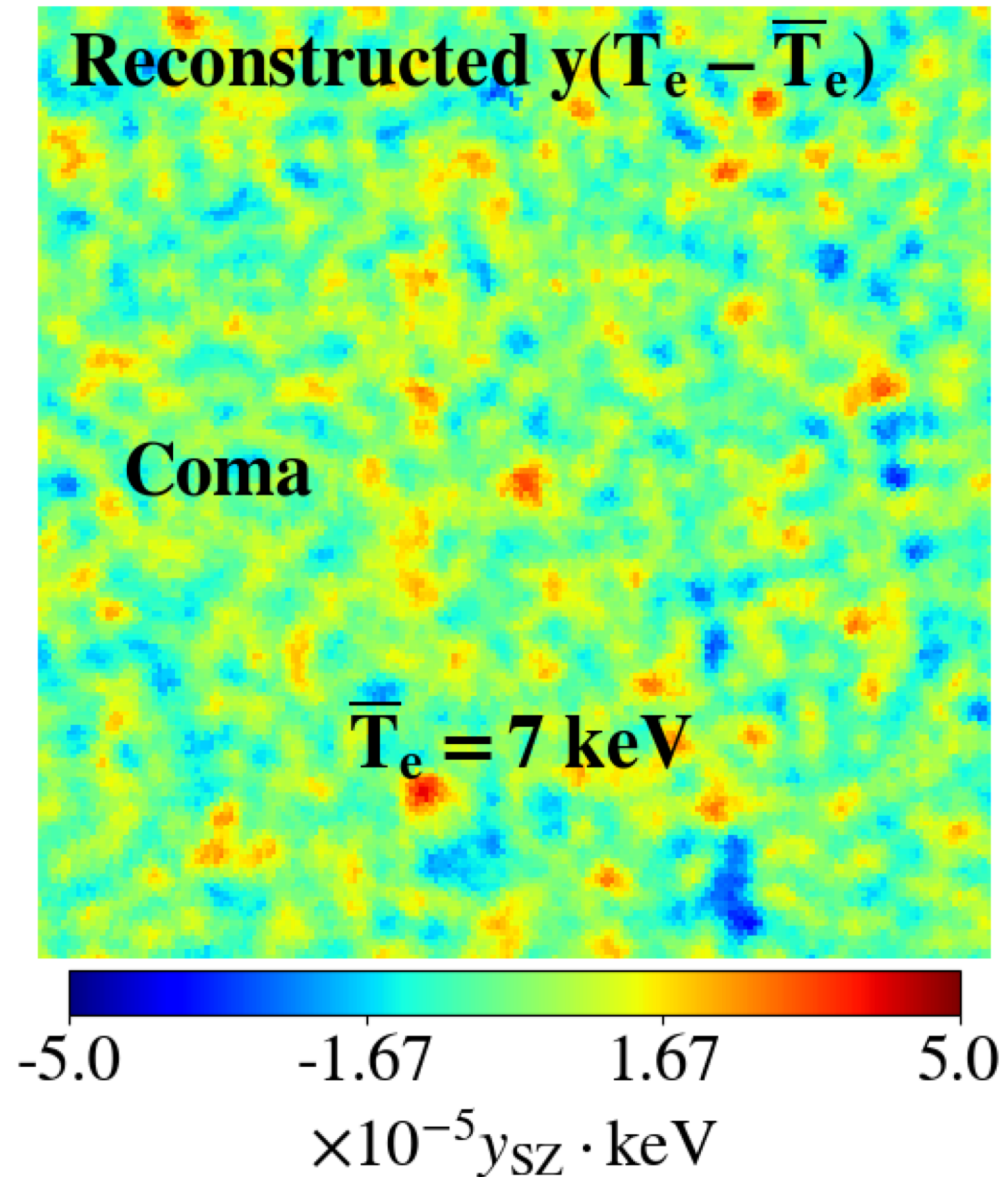
- Decrement if actual temperature $T_e < \bar{T}_e$
- Increment if actual temperature $T_e > \bar{T}_e$
- Null if actual temperature $T_e \simeq \bar{T}_e$

Cluster spectroscopy across temperature



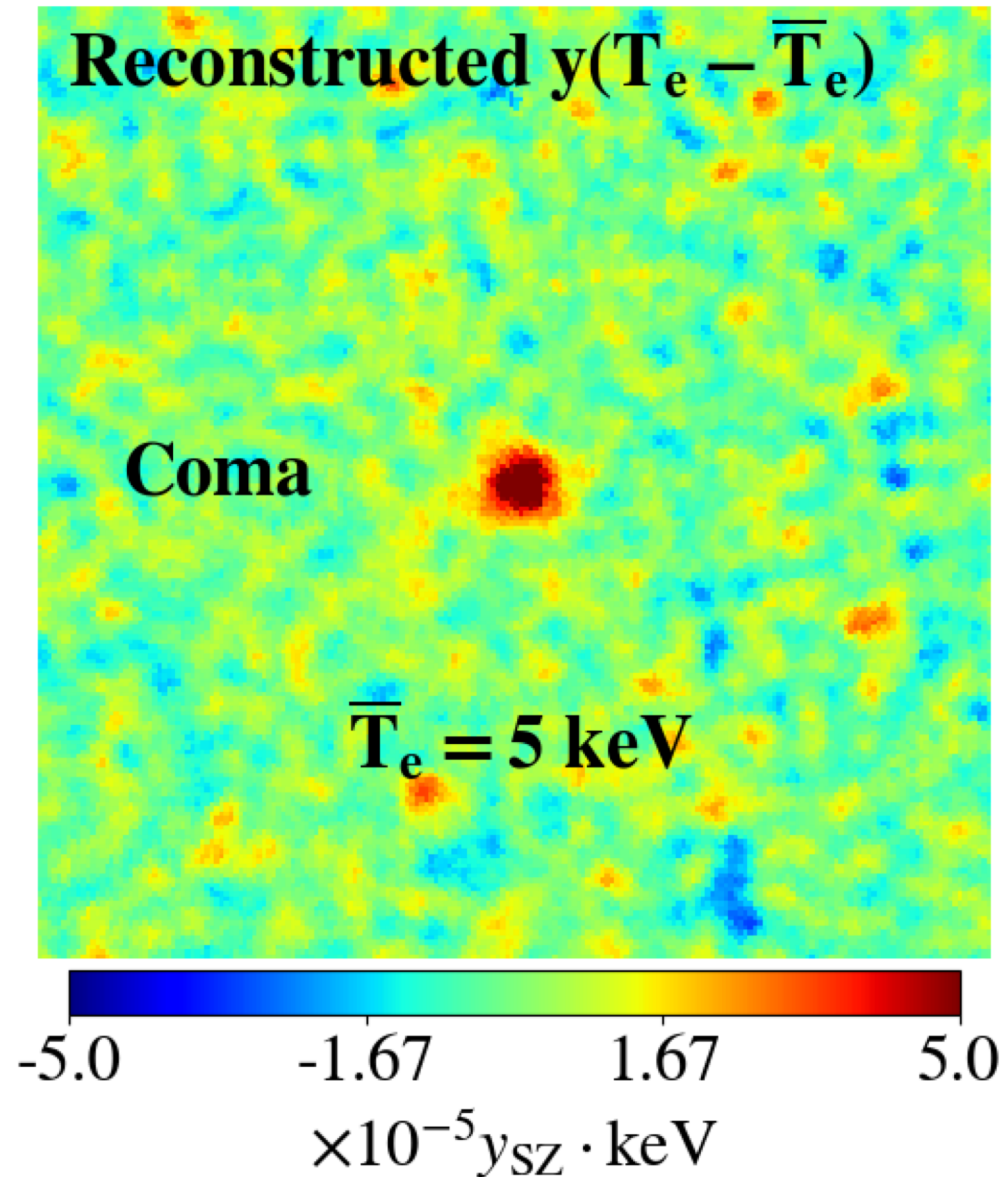
$$\bar{T}_e = 10 \text{ keV}$$

Cluster spectroscopy across temperature



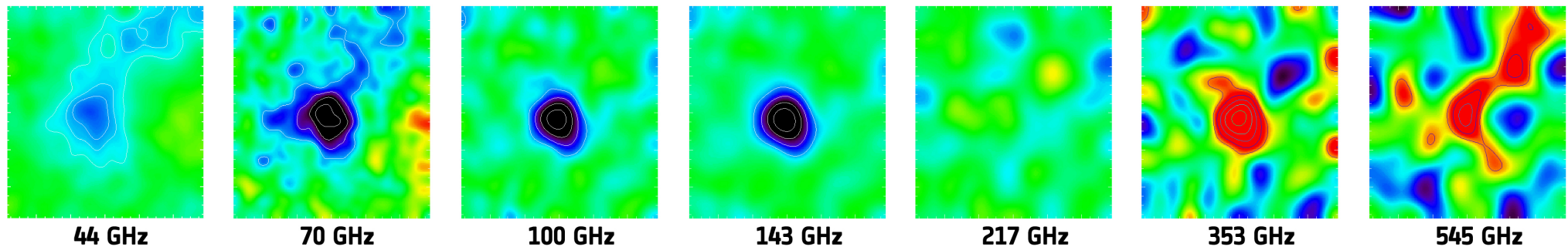
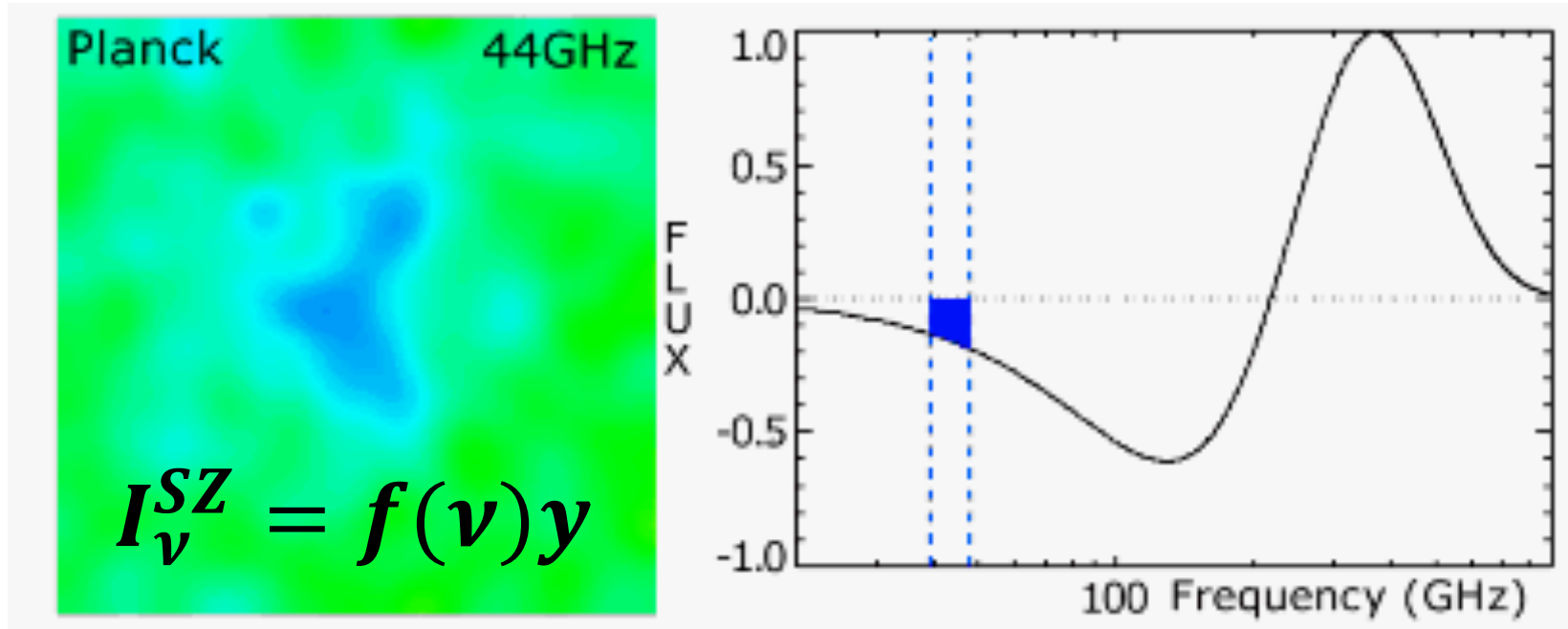
$$\bar{T}_e = 7 \text{ keV}$$

Cluster spectroscopy across temperature



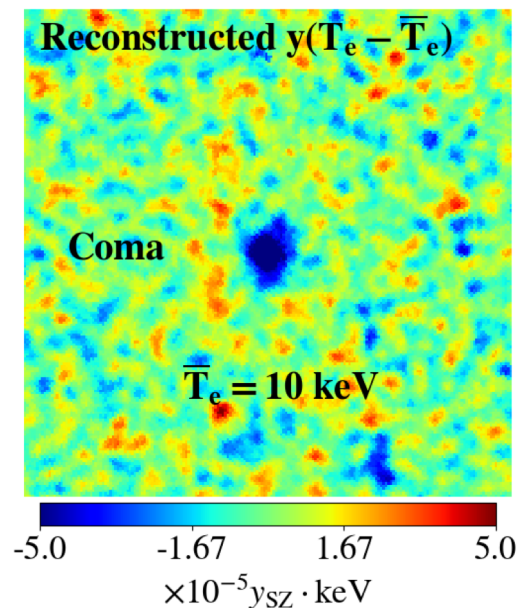
$$\bar{T}_e = 5 \text{ keV}$$

First SZ revolution (2010-2020): Cluster spectroscopy across frequency

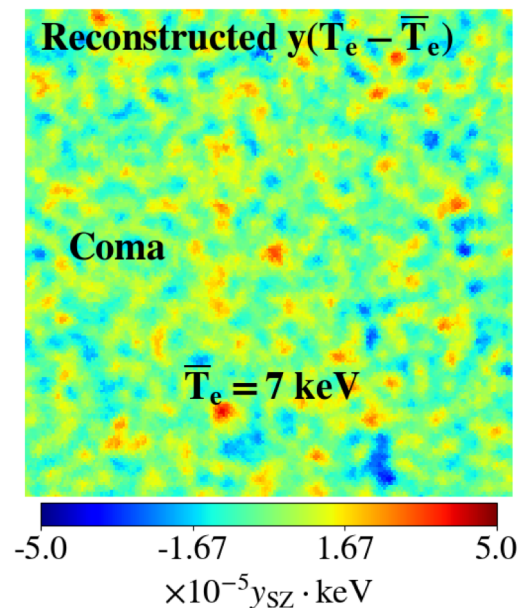


Next SZ revolution (2020-2030): Cluster spectroscopy across temperature

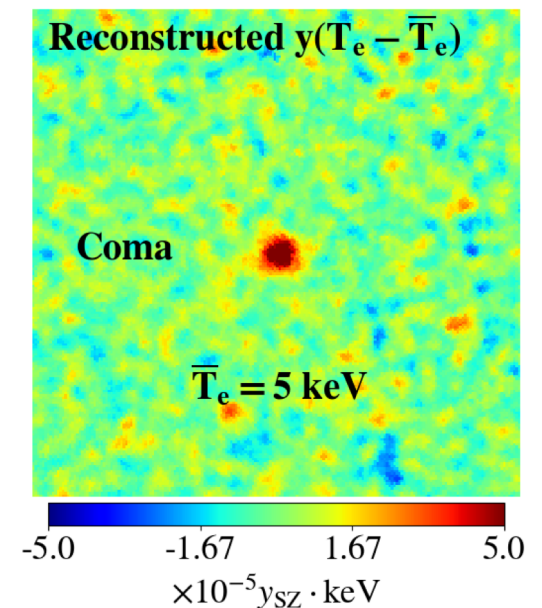
Coma



colder than 10 keV



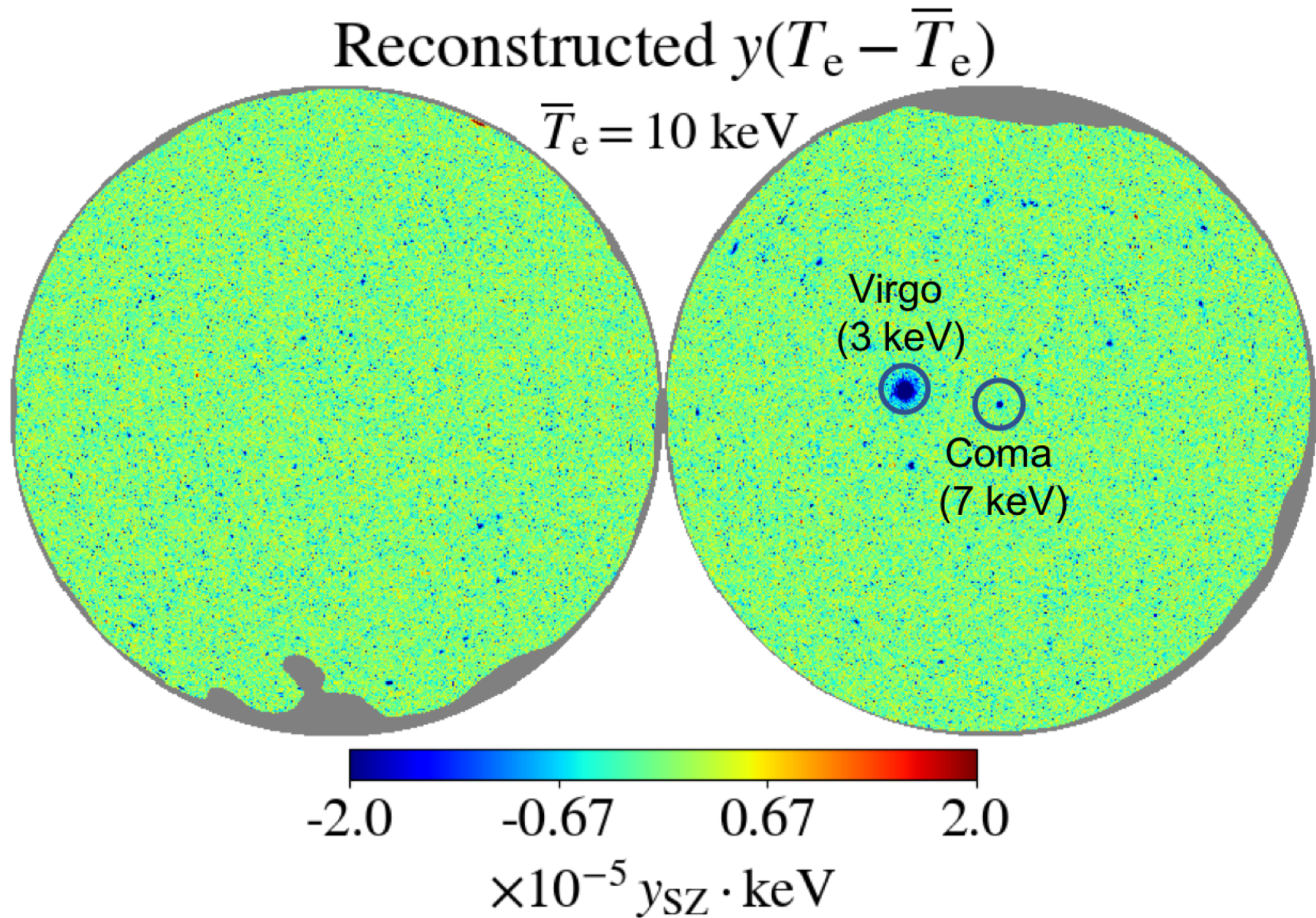
closer to 7 keV



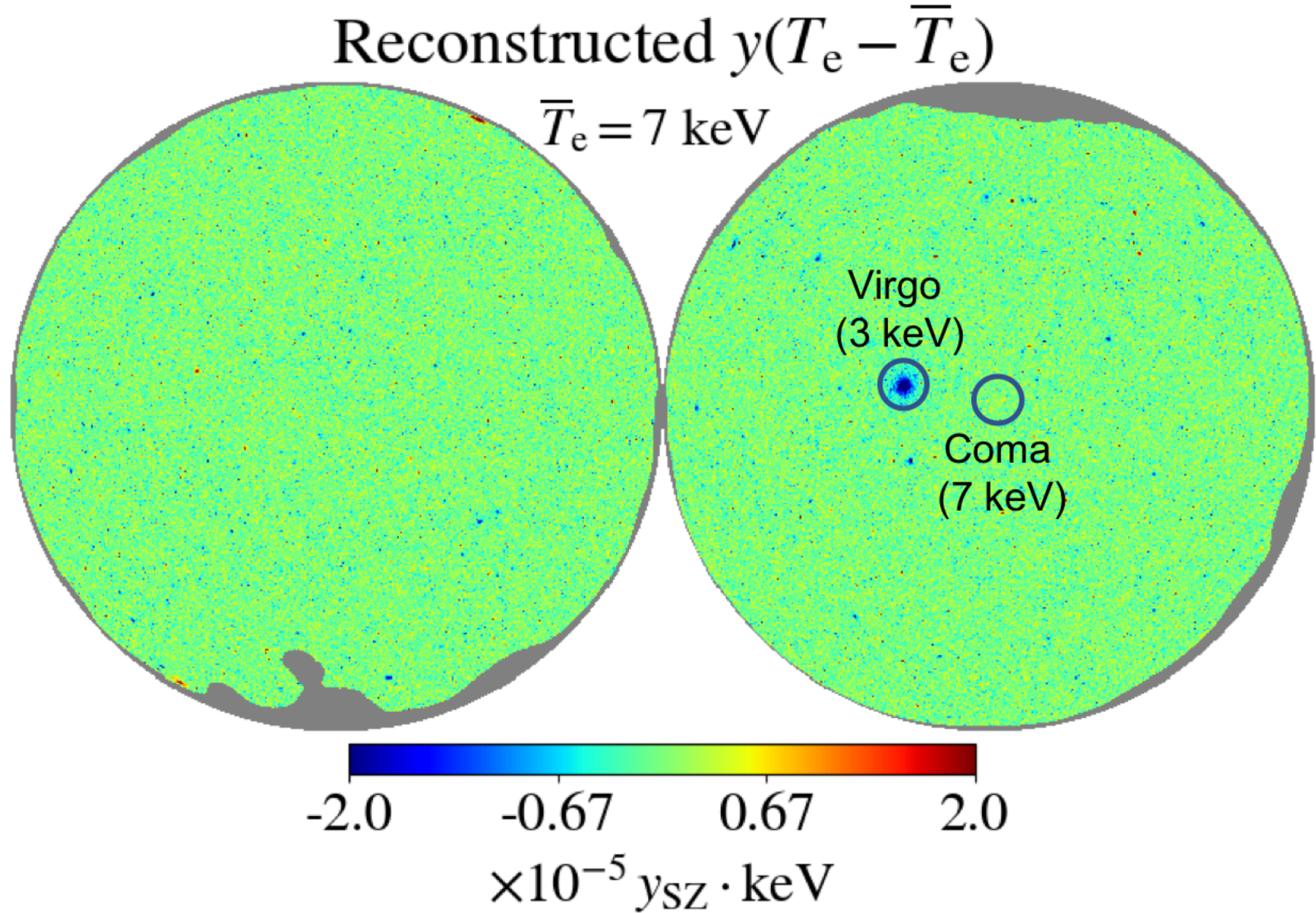
hotter than 5 keV

$$y(T_e - \bar{T}_e)$$

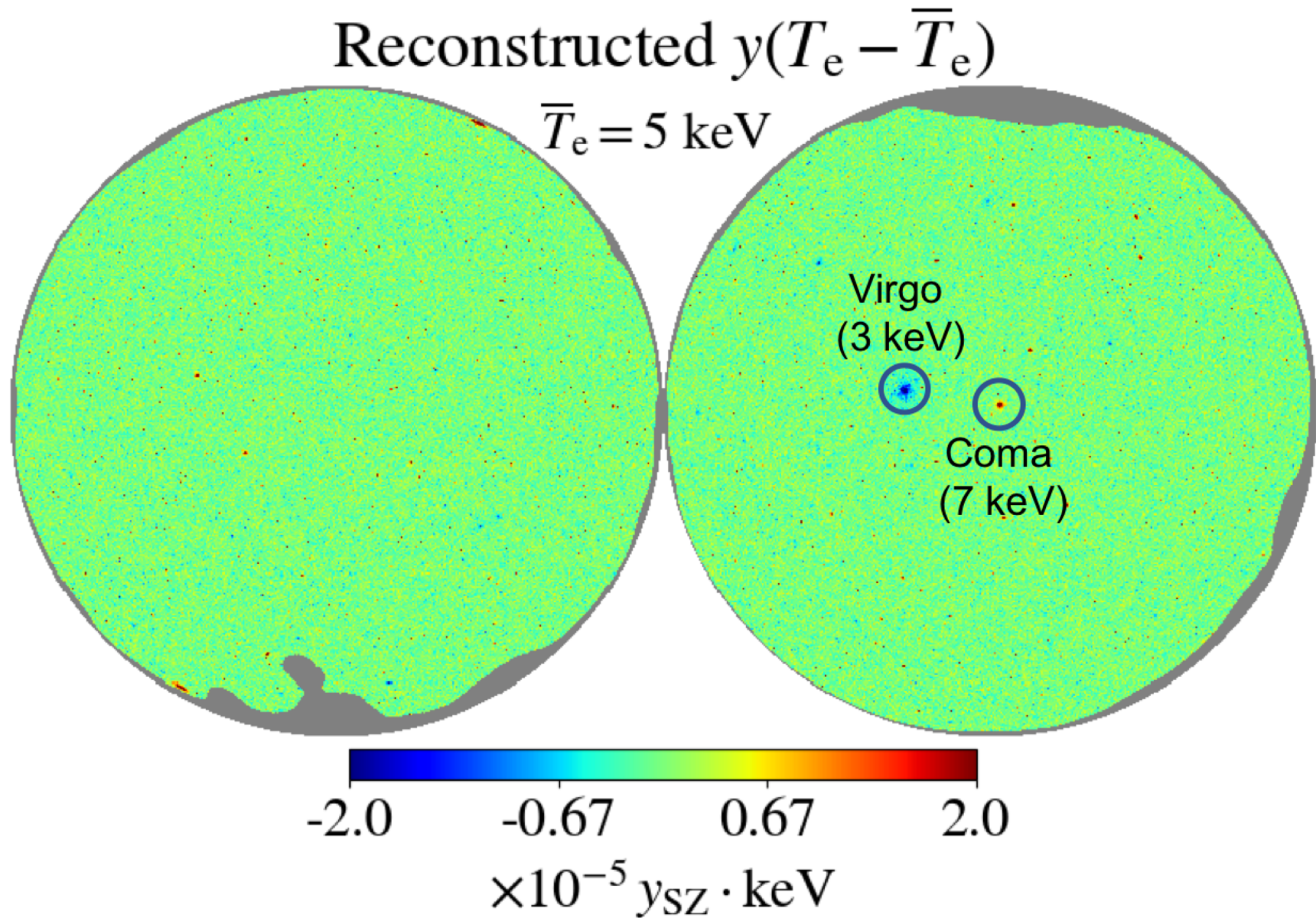
Full-sky temperature spectroscopy



Full-sky temperature spectroscopy



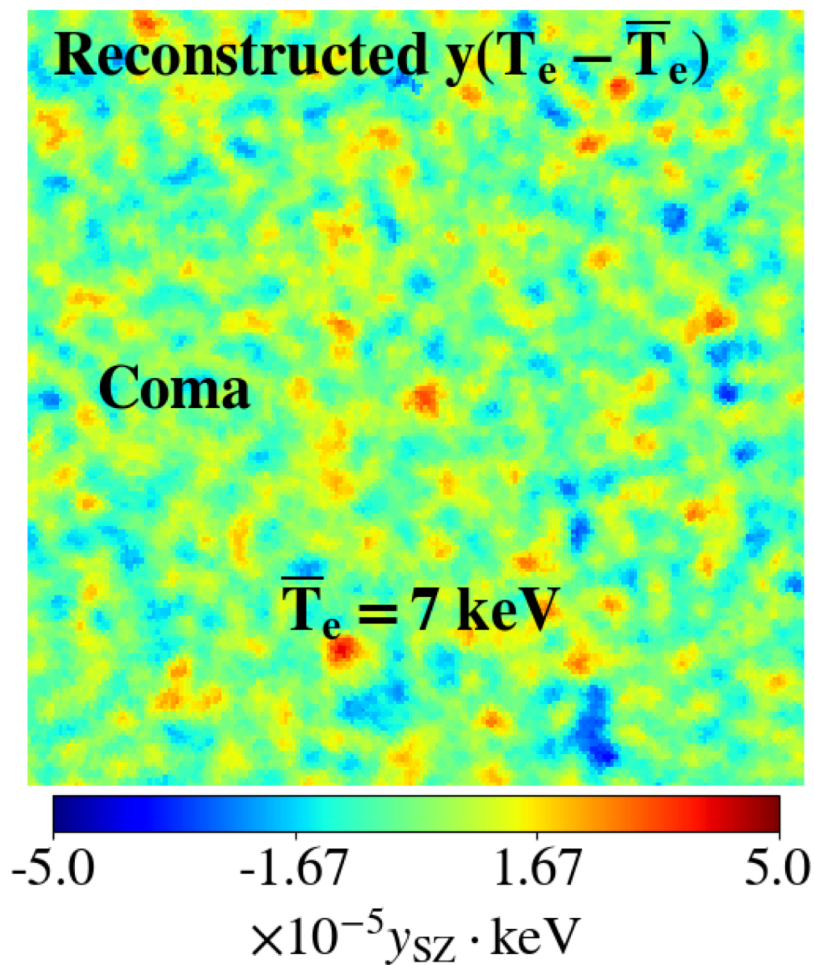
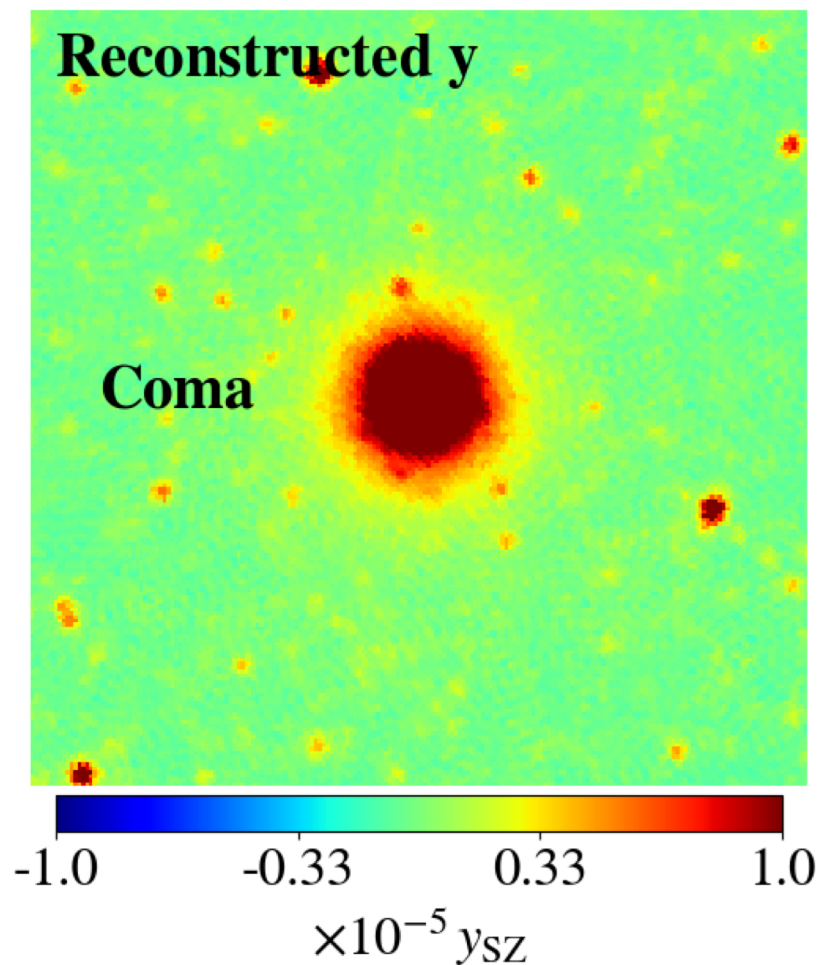
Full-sky temperature spectroscopy



Reconstructed rSZ components:

y

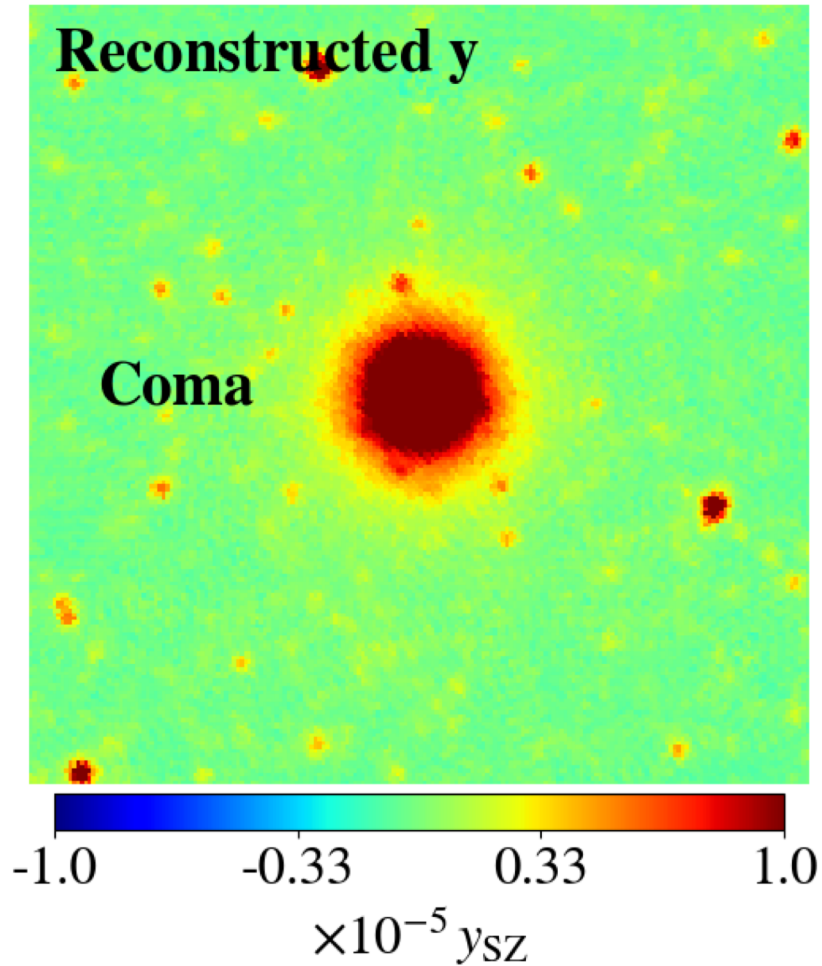
$$y\Delta T_e \equiv y(T_e - \bar{T}_e)$$



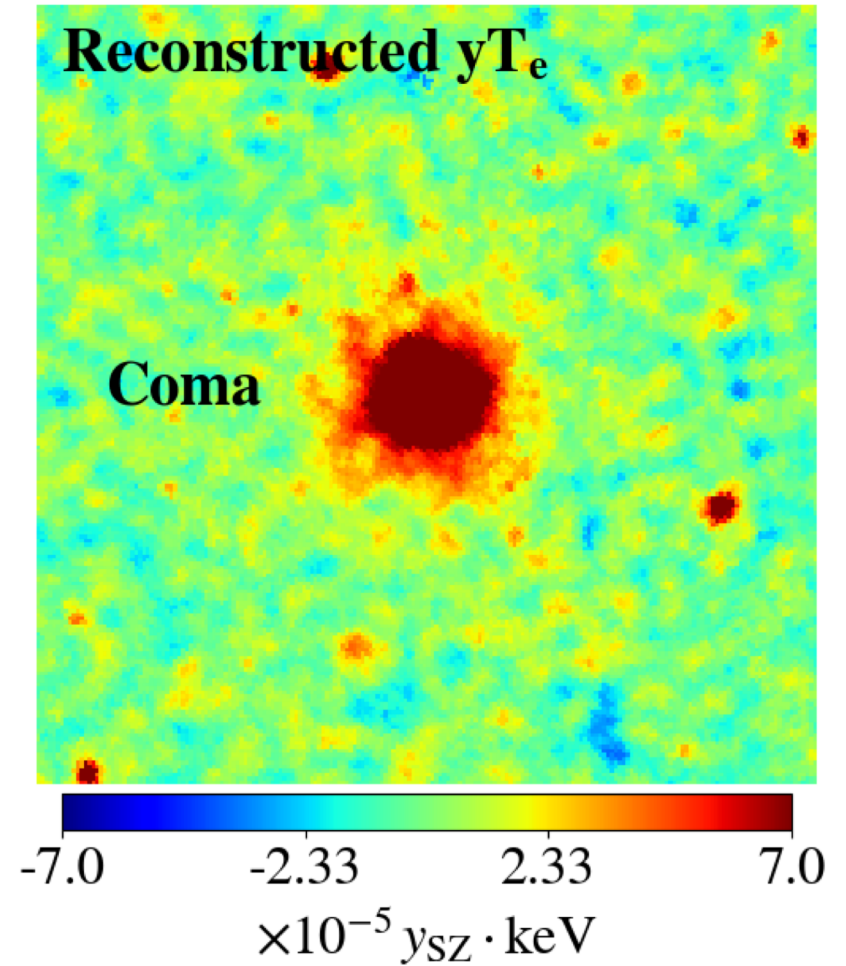
$$\Rightarrow yT_e = y\Delta T_e + \bar{T}_e y$$

Mapping y and yT_e components

y

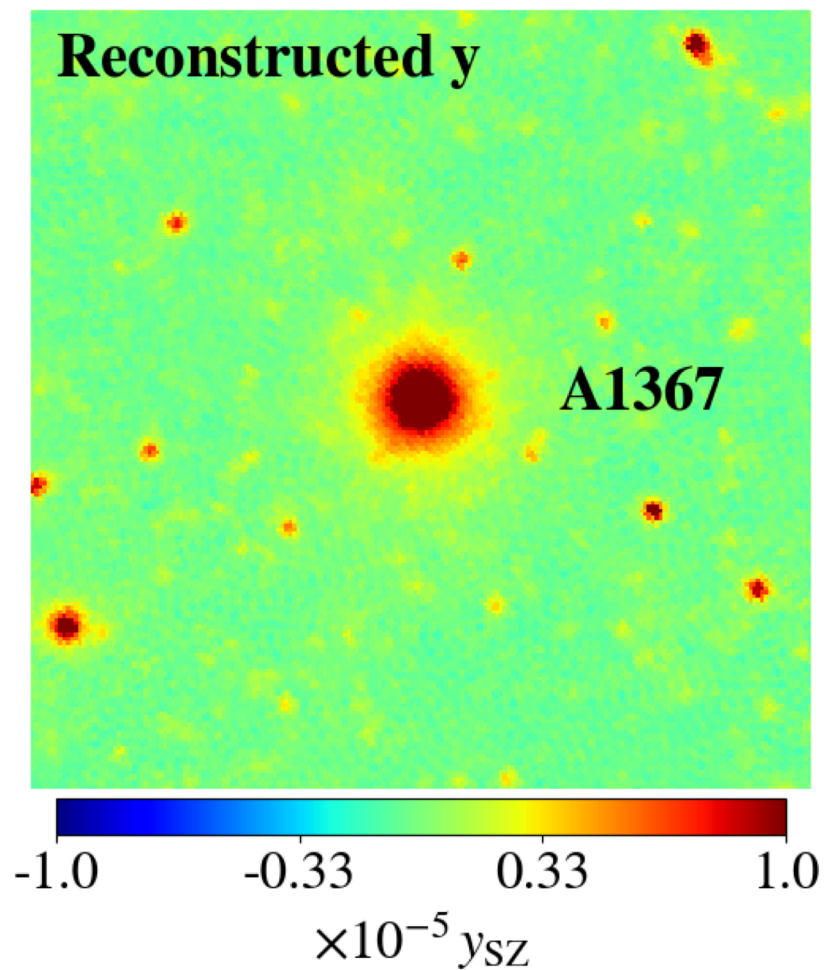


yT_e

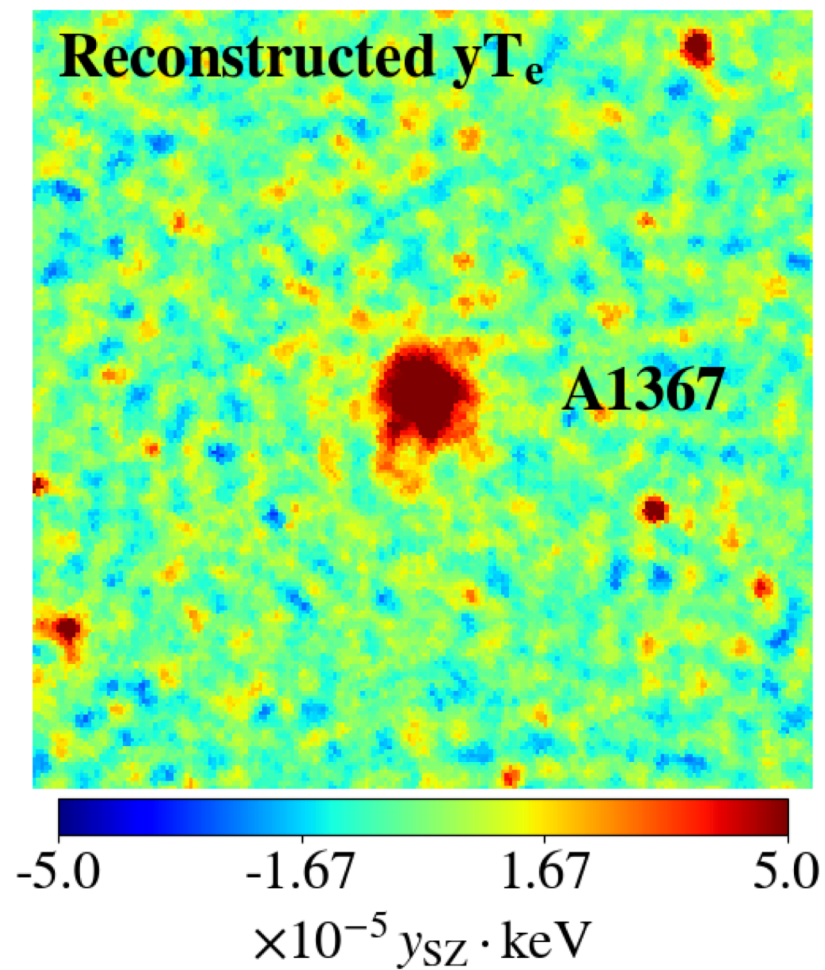


Mapping y and yT_e components

y



yT_e

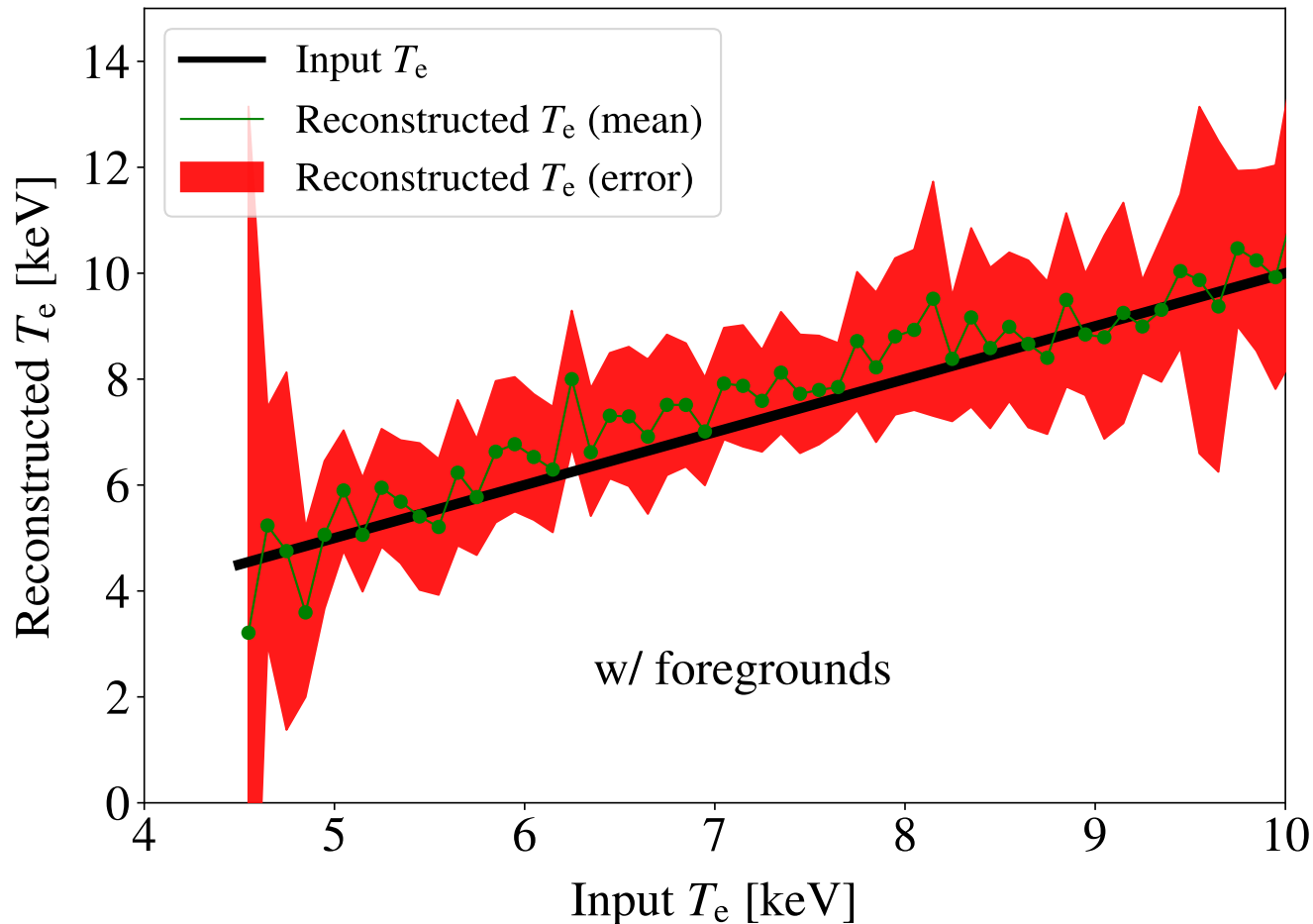


Mapping cluster temperatures T_e across the entire sky

$$T_e^y[R_{500}] = \frac{\langle (yT_e)(\vec{\theta}) \rangle_{|\vec{\theta}-\vec{\theta}_c| \leq R_{500}}}{\langle y(r) \rangle_{|\vec{\theta}-\vec{\theta}_c| \leq R_{500}}}$$

y -weighted average cluster temperature over R_{500}

Recovered electron temperatures T_e of clusters across the entire sky



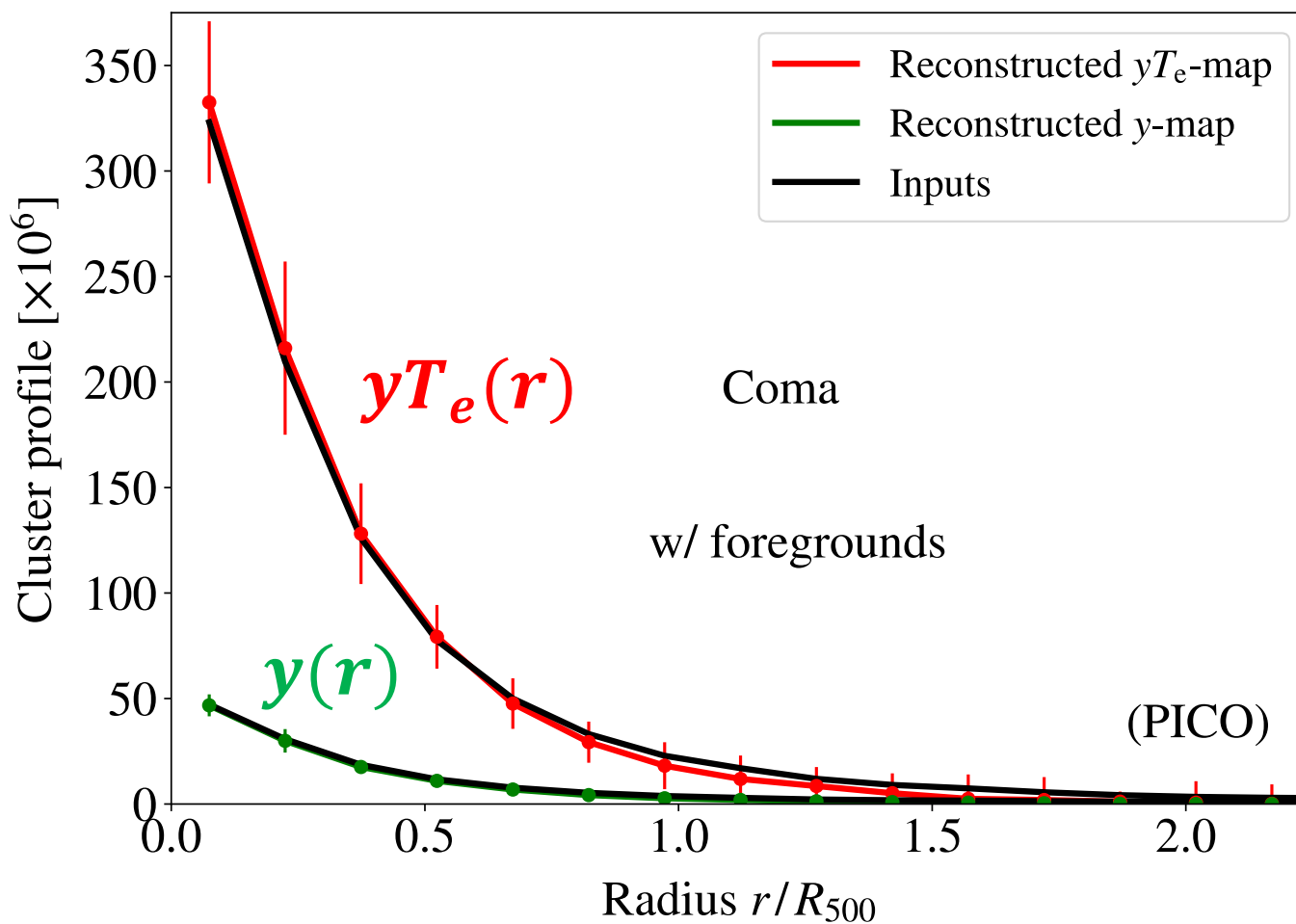
The recovered rSZ temperatures offer a new proxy for determining cluster masses without relying on X-rays

Temperature profile of a cluster

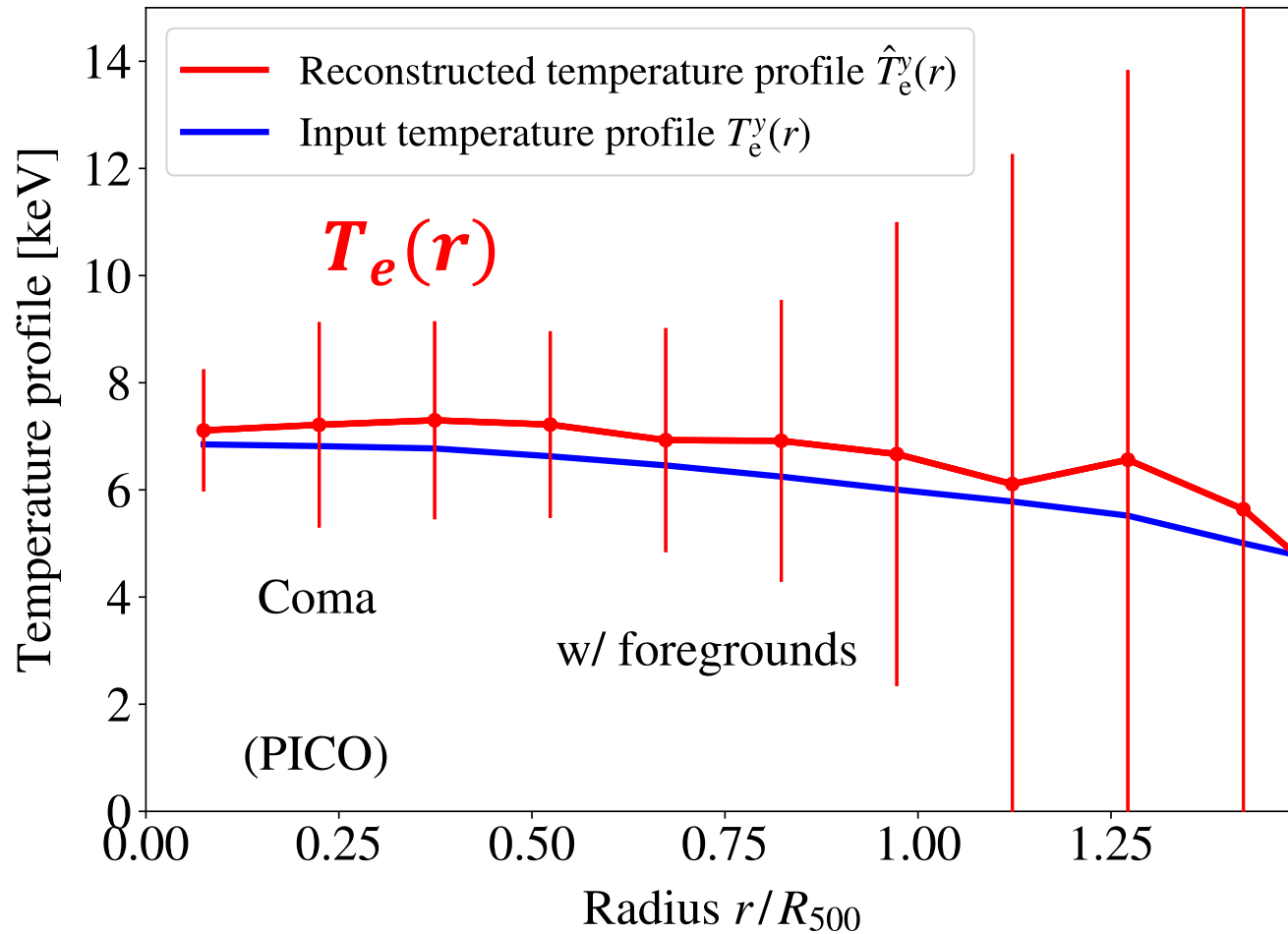
$$T_e^y(r) = \frac{(yT_e)(r)}{y(r)}$$

y -weighted temperature profile

Reconstructed cluster profiles



Reconstructed cluster temperature profile



$$T_e = (7.1 \pm 0.69) \text{ keV}$$

10 σ measurement with PICO

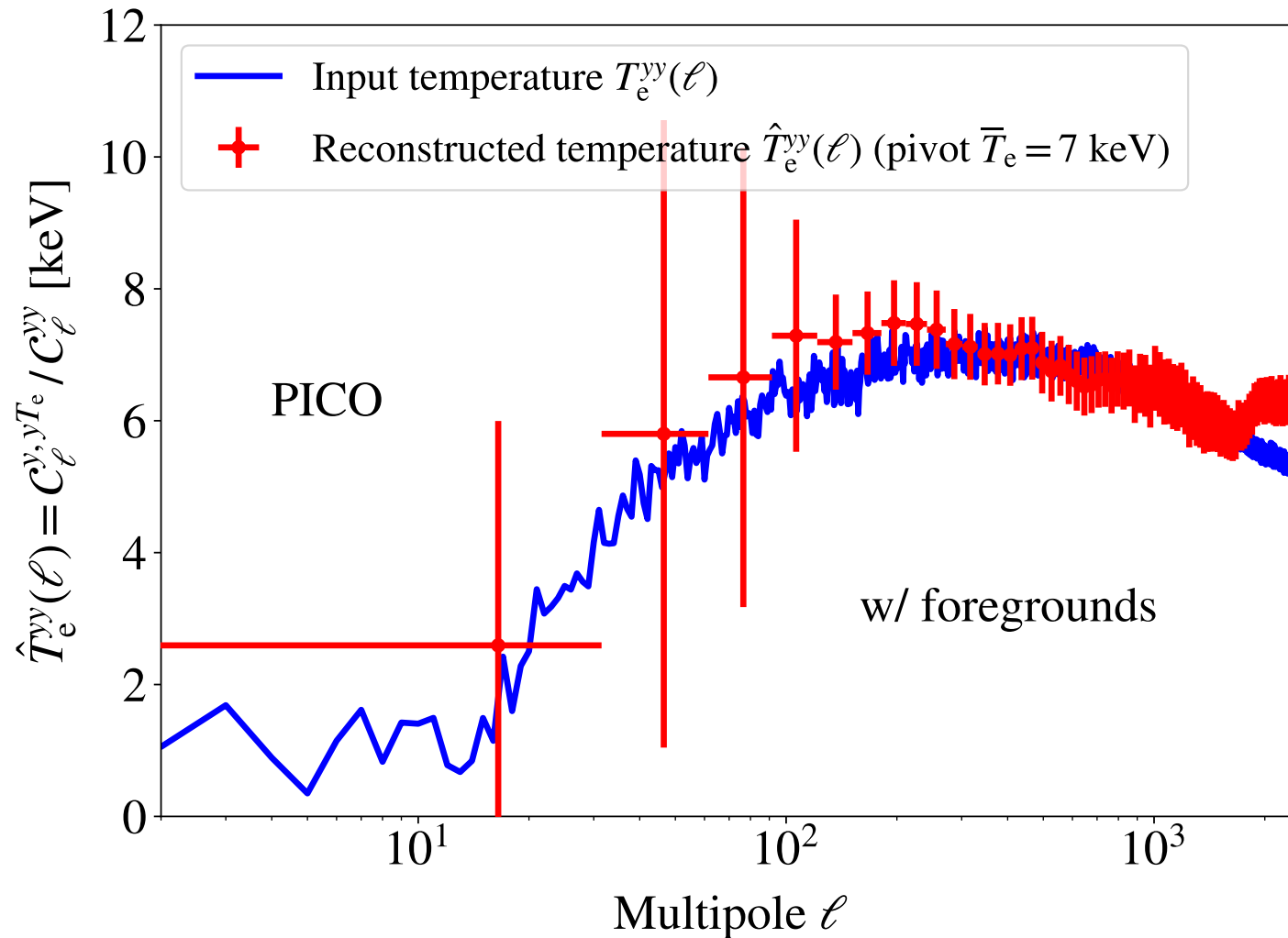
Electron temperature power spectrum

$$T_e^{yy}(\ell) = \frac{\langle (yT_e)_{\ell m} y_{\ell m}^* \rangle}{\langle y_{\ell m} y_{\ell m}^* \rangle} = \frac{C_\ell^{y,yT_e}}{C_\ell^{yy}}$$

y^2 -weighted average temperature over the full sky
across angular scales

*Ratio of the cross-power spectrum between the y - and yT_e -maps
and the auto-power spectrum of the y -map*

Electron temperature power spectrum $T_e^{yy}(\ell)$

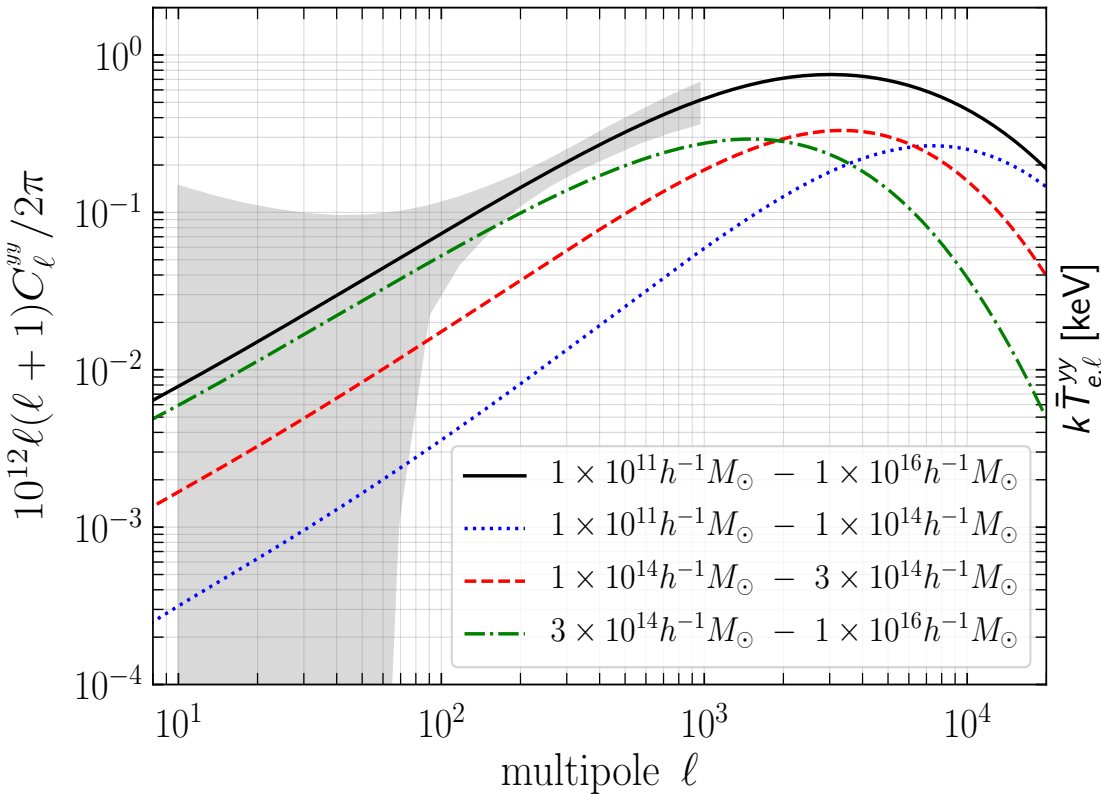


$T_e^{yy}(\ell)$ provides a new map-based observable, complementing the y -map power spectrum C_{ℓ}^{yy} , to constrain cosmological parameters!

Two observables for cluster cosmology !

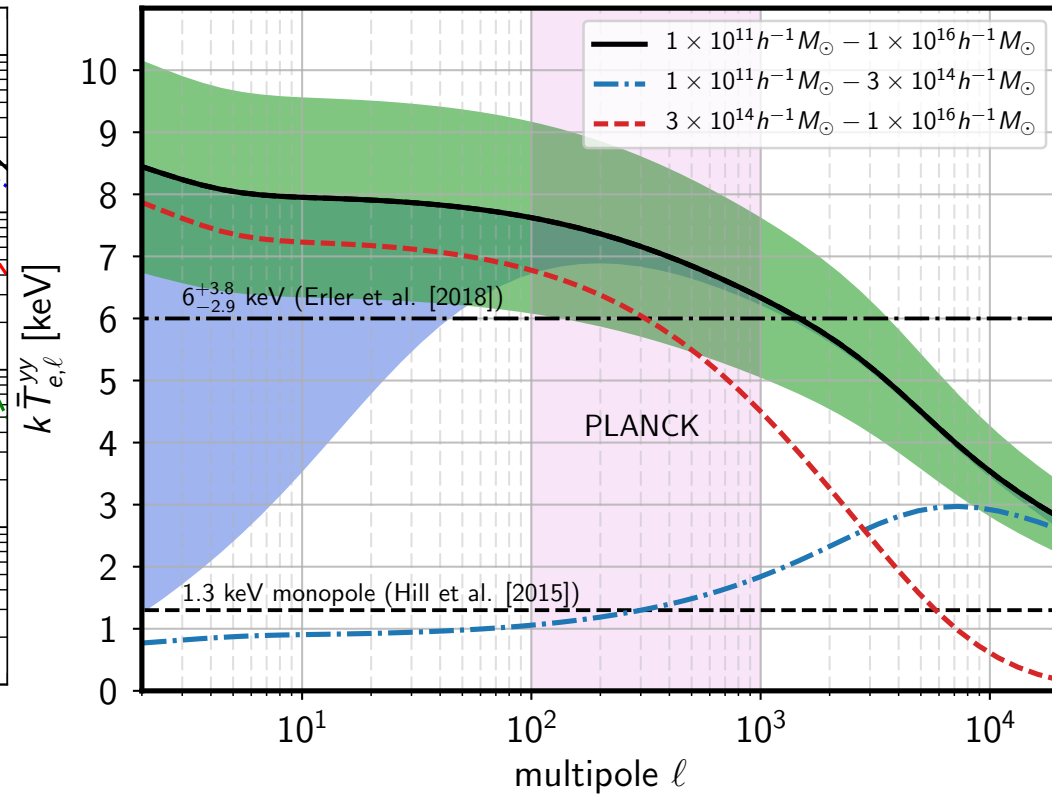
y -power spectrum:

$$C_{\ell}^{yy}$$



T_e -power spectrum:

$$T_e^{yy}(\ell) = C_{\ell}^{T_e y, y} / C_{\ell}^{yy}$$



The shapes of power spectra C_{ℓ}^{yy} and $T_e^{yy}(\ell)$ have different scaling with cosmological parameters

Cosmology with rSZ temperature

$$C_{\ell}^{yy} = \int_{z_{\min}}^{z_{\max}} dz \frac{dV}{dz d\Omega} \int_{M_{\min}}^{M_{\max}} dM \underbrace{\frac{dn(M, z)}{dM}}_{\text{halo mass function}} \underbrace{|y_{\ell}(M, z)|^2}_{\text{cluster pressure profile}}$$

different
scaling
with σ_8

$$C_{\ell}^{y, yT_e} = \int_{z_{\min}}^{z_{\max}} dz \frac{dV}{dz d\Omega} \int_{M_{\min}}^{M_{\max}} dM \underbrace{\frac{dn(M, z)}{dM} T_e(M)}_{\text{Effective "halo mass function"}} \underbrace{|y_{\ell}(M, z)|^2}_{\text{cluster pressure profile}}$$

same
scaling
with
mass-bias b

$$T_e^{yy}(\ell) = \frac{C_{\ell}^{y, yT_e}}{C_{\ell}^{yy}} \text{ depends only on } \sigma_8 !$$

Two independent map-based observables for future cluster cosmology

The shapes of the power spectra $T_e^{yy}(\ell)$ and C_ℓ^{yy} have different scaling with cosmological parameters

C_ℓ^{yy} depends on σ_8 and “mass-bias” b in a degenerate form
 $T_e^{yy}(\ell)$ depends on σ_8 but is insensitive to b

$T_e^{yy}(\ell)$ will allow to break parameter degeneracies, possibly alleviating some of the current tensions on cosmological parameters

Conclusions

- ✓ New component separation method to disentangle the y and T_e observables of the rSZ effect
- ✓ High frequencies > 300 GHz are crucial for rSZ temperature measurements:
MILLIMETRON, PICO, Voyage 2050 BACKLIGHT missions would be of great value!
- ✓ A PICO-type mission allows us to map the rSZ temperatures of clusters across the entire sky, thus offering a new proxy for determining cluster masses
- ✓ A PICO-type mission allows us to reconstruct the temperature profiles of individual clusters, thus offering a complete understanding of the thermodynamical properties of clusters
- ✓ We envision a second SZ revolution for 2020-2030:
 - A T_e -map along with the y -map!
 - Cluster spectroscopy across temperatures
 - The electron-temperature power spectrum $T_e^{yy}(\ell)$ will deliver a new map-based observable, complementing C_ℓ^{yy} , to constrain cosmological parameters!

Thank you!

Backup slides

rSZ temperature vs X-ray temperature

- ✓ To date, SZ studies rely on X-rays measurements for cluster temperatures
Arnaud et al, A&A (2010)
- ✓ However, X-ray temperatures are not be the best proxy for measuring virial temperatures of galaxy clusters:
“X-rays trace the central denser component of the cluster unlike SZ, while the cluster gas is not homogeneous and isothermal”
Pointecouteau et al, A&A (1998)
- ✓ N-body/hydrodynamic simulations predict that:
“The SZ temperature is hotter than the X-ray temperature and is within 10 per cent of the virial temperature of the cluster”
Kay et al, MNRAS (2008)

Mapping cluster Compton- γ across the sky

$$d(\nu, \vec{\theta}) = \underbrace{f(\nu, T_e = \bar{T}_e)}_{\text{energy spectrum of first component}} \underbrace{y(\vec{\theta})}_{\text{fluctuations of first component}} + \underbrace{\frac{\partial f(\nu, T_e = \bar{T}_e)}{\partial T_e}}_{\text{energy spectrum of second component}} \underbrace{(T_e(\vec{\theta}) - \bar{T}_e)y(\vec{\theta})}_{s(\vec{\theta}; T_e) \text{ fluctuations of second component}} + \underbrace{n(\nu, \vec{\theta})}_{\text{foregrounds + noise}}$$

- Component separation with "Constrained-ILC" ([Remazeilles et al MNRAS 2011](#)):

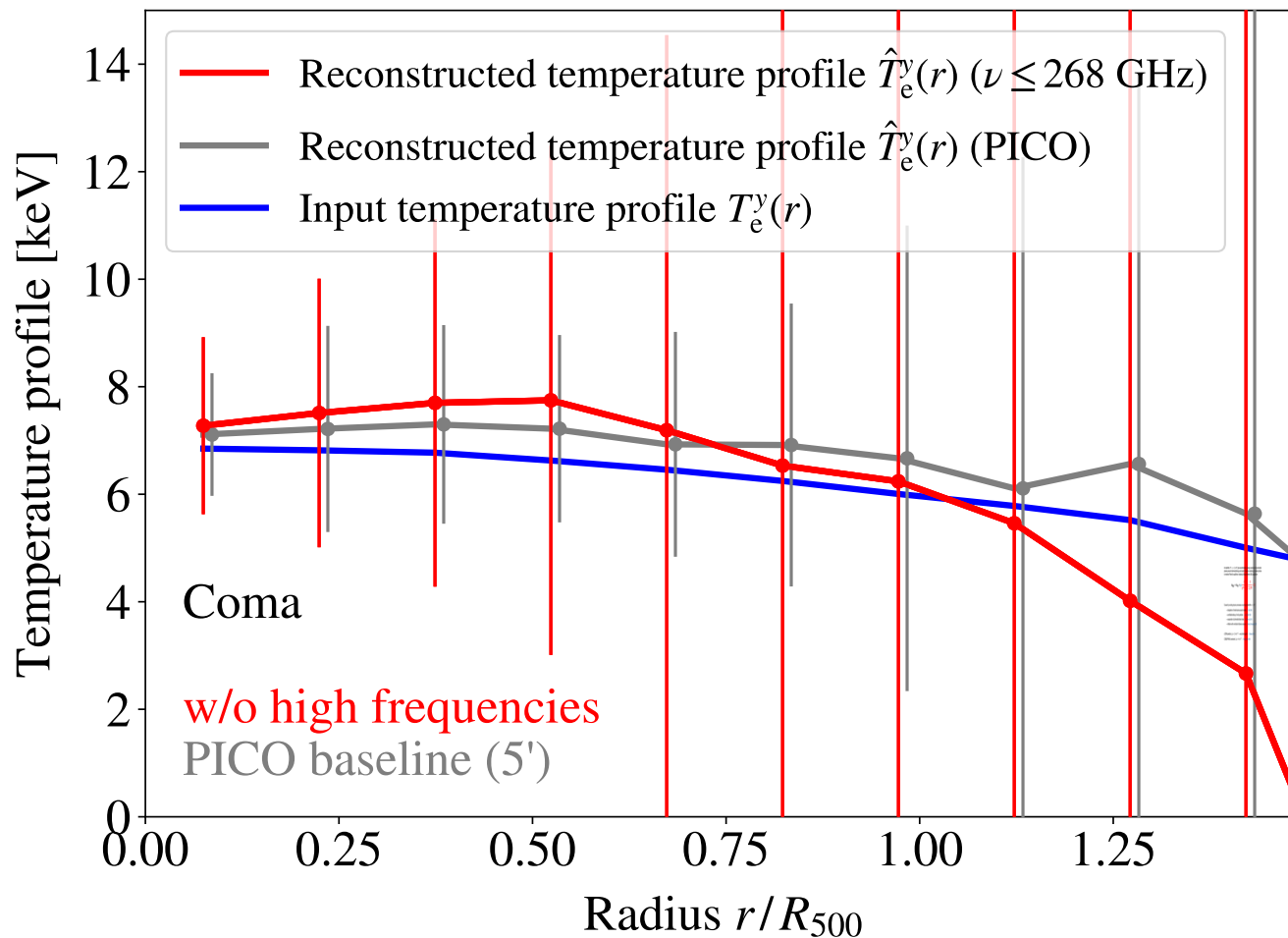
$$\hat{y}(\vec{\theta}; T_e) = \sum_{\nu} w(\nu) d(\nu, \vec{\theta}) \quad \text{such that} \quad \begin{cases} \langle \hat{y}^2 \rangle = \mathbf{w}^t \langle d d^t \rangle \mathbf{w} \text{ minimum} \\ \sum_{\nu} w(\nu) \frac{\partial f(\nu, \bar{T}_e)}{\partial T_e} = 0 \\ \sum_{\nu} w(\nu) f(\nu, \bar{T}_e) = 1 \end{cases}$$

- Analytic solution: $\mathbf{w}^t = \frac{(\partial_{T_e} \mathbf{Y}^t \mathbf{C}^{-1} \partial_{T_e} \mathbf{Y}) \mathbf{Y}^t \mathbf{C}^{-1} - (\mathbf{Y}^t \mathbf{C}^{-1} \mathbf{Y}) \partial_{T_e} \mathbf{Y}^t \mathbf{C}^{-1}}{(\partial_{T_e} \mathbf{Y}^t \mathbf{C}^{-1} \partial_{T_e} \mathbf{Y})(\mathbf{Y}^t \mathbf{C}^{-1} \mathbf{Y}) - (\partial_{T_e} \mathbf{Y}^t \mathbf{C}^{-1} \mathbf{Y})^2} \quad (\mathbf{C} \equiv \langle d d^t \rangle) \quad \text{yields to:}$

$$\hat{y}(\vec{\theta}) = y(\vec{\theta}) + \mathbf{w}^t \mathbf{n}$$

Compton- γ map over the sky

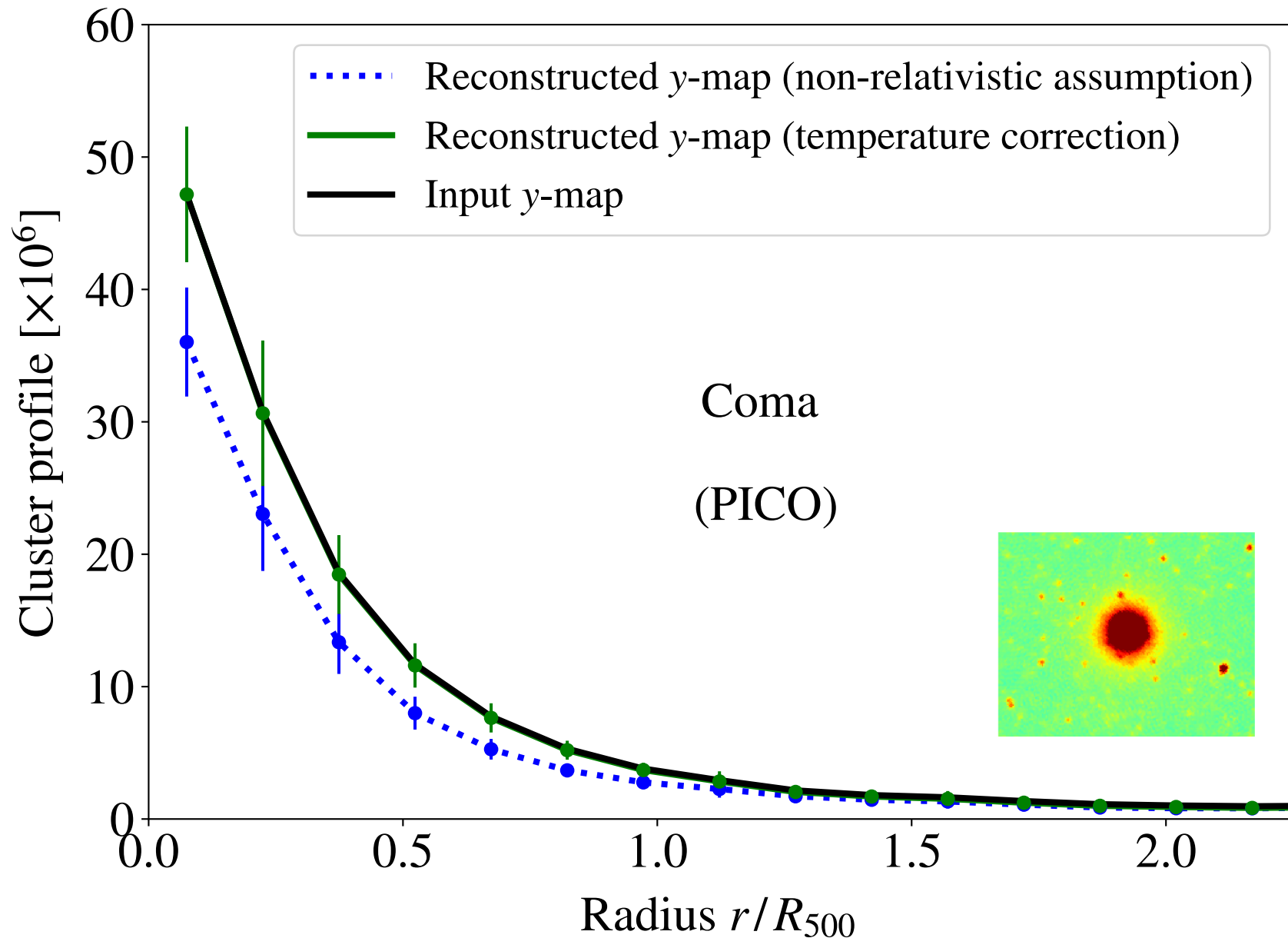
Importance of high frequencies $\gtrsim 300$ GHz



✓ With high frequencies: $T_e = (7.1 \pm 0.69)$ keV

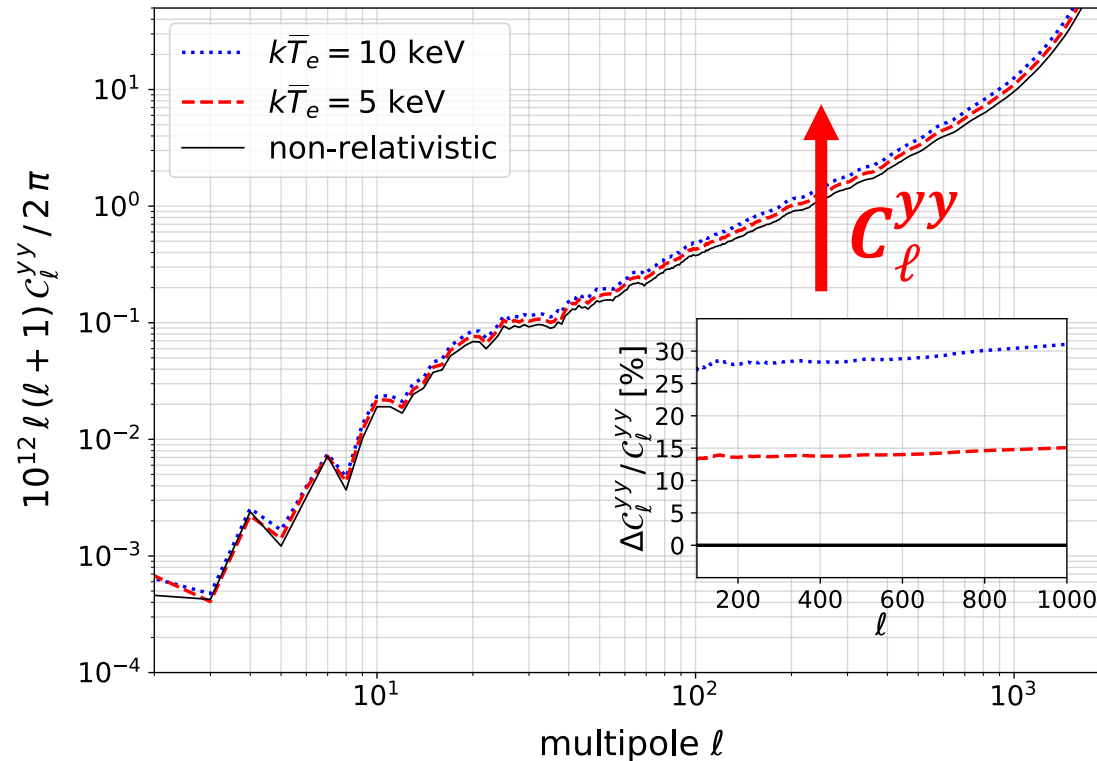
✓ Without high frequencies: $T_e = (7.4 \pm 1.2)$ keV

Impact of neglecting relativistic temperature corrections on cluster pressure profiles



Relativistic temperature corrections to the Planck SZ power spectrum

Remazeilles, Bolliet, Rotti, Chluba, MNRAS 2019



C_ℓ^{yy} increases with average cluster temperature $\bar{T}_e$

$$C_\ell^{yy} \propto \sigma_8^{8.1} \Rightarrow \frac{\Delta \sigma_8}{\sigma_8} \simeq 0.019 \left(\frac{k\bar{T}_e}{5 \text{ keV}} \right)$$

Reduces Planck's tension by 1σ for $\bar{T}_e \simeq 5 \text{ keV}$